

FRICTION FACTORS FOR BED FORMS AND
VERTICAL OBSTRUCTIONS IN COASTAL FLOODING

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LIST OF SYMBOLS

A	cross-sectional area
A	projected area of obstruction (Chap. I)
A	experimental coefficient (Chap. V)
A _D	equivalent drag area
A _L	lateral area
A _o	cross-sectional area of obstructions
a	one-half the bed form height
B	width of water surface (Chap. II)
B	experimental coefficient (Chap. V)
B	width of flow region
B _S	modified width of flow region
b	flow width in flume
b'	modified flow width in flume
C	experimental exponent
C _D	drag coefficient
C _{De}	drag coefficient for equivalent obstruction
C _{Dm}	mean value of drag coefficient
C _{Dn}	drag coefficient for natural obstruction
C _{Dnm}	mean value of drag coefficient for natural obstruction
C _e	shape factor for equivalent obstruction
C _n	shape factor for natural obstruction
D	pipe diameter
d	depth in open channel
$\bar{d}$	two point depth average
d _c	depth in contracted section
d _{cr}	critical depth
d _f	flume depth
d _L	depth of flow for larger bed forms
d _m	mean depth
d _s	depth of flow for smaller bed forms
d ₅₀	median grain size diameter
e	2.71828. . . base of natural logarithms
F	$(1 - \frac{\epsilon}{d} - \frac{a}{d})$
F _D	drag force on obstruction
F _e	total force in equivalent system
F _n	total force in natural system
Fr	Froude number

Fr_m	Froude number based on mean depth
f	friction factor for pipe flow
f'	friction factor based on hydraulic radius
f'_b	friction factor for bed forms
f'_c	combined friction factor
f'_D	friction factor for drag forces
f'_s	friction factor for skin friction
f'_T	theoretical friction factor
g	acceleration due to gravity
H	head on weir
H_b	concrete block spacing function
H_c	spacing function
H_1	fractional spacing function
h	piezometric head
h_3	piezometric head difference
i	index of summation
K	convergence factor
K	experimental constant (Chap. V)
K_A	d'Aubuisson's coefficient
K_B	combined coefficient
K_N	Nagler's coefficient
K_Y	Yarnell's coefficient
k	equivalent sand roughness
L	length of flow region
L_b	length of bed form (Chap. III)
L	pipe length (Chap. II)
L	side length of flow area (Chap. III)
$\ln$	natural logarithm
M	Manning's resistance coefficient
m	number of rows of obstruction
N	total number of obstructions
N_e	number of obstructions in equivalent system
N_n	number of obstructions in natural system
n	number of obstructions per row
P	wetted perimeter
P_D	apparent obstruction density
p_o	pressure at the centroid (Chap. II)
P_o	dimensionless pipe density
$P.G.$	point gauge reading

Q	discharge
q	discharge per unit width
R	hydraulic radius
Re	Reynolds number
Re_δ	Reynolds number based on characteristic dimension
$SEGL$	slope of the energy grade line
S_l	longitudinal spacing
S_t	transverse spacing
T	temperature
v	free stream velocity (Chap. II)
v	point velocity (Chap. IV)
v_m	mean velocity
v_{m25}	mean velocity for 25% depth profile
v_{max25}	maximum velocity for 25% depth profile
v_s	spatially averaged mean velocity
x	position coordinate along channel
y	flow depth in bridge pier experiments
z	bed elevation
α	energy coefficient
α	ratio of pier width to channel width (Chap. V)
α'	momentum coefficient
α_{cr}	energy coefficient at critical depth
α_{crest}	energy coefficient at bed form crest
β	bed inclination with horizontal
β	velocity coefficient (Chap. V)
γ	unit weight of water
ΔH	energy loss
ΔH_b	energy loss due to bed forms
ΔH_s	energy loss due to skin friction
ΔH_{SE}	energy loss due to a sudden expansion
ΔH_v	energy loss due to vertical obstructions
δ	characteristic dimension of obstruction
δ_e	characteristic dimension of equivalent obstruction
δ_m	mean value of characteristic dimension
δ_n	characteristic dimension of natural obstruction
δ_{nm}	mean value of characteristic dimension for natural obstruction
δ'_n	modified characteristic dimension in natural system
δ'_{nm}	mean value of modified characteristic dimension in natural system
ϵ	water surface depression (Chap. III)

ζ_{SE}	sudden expansion energy loss coefficient
η	coefficient ratio
θ	adjustment factor for Nagler equation
λ	distribution factor for vertical obstructions
ν	kinematic viscosity
π	constant, 3.14159 . . .
ρ	fluid density
σ	conveyance ratio
σ_{on}^2	variance of modified characteristic dimension distribution
τ_o	equivalent obstruction shear stress
τ_o	average bed shear stress
τ_{oe}	shear stress in equivalent system
τ_{on}	shear stress in natural system
Φ	distribution function (Chap.III)
ϕ	dimensionless numerical coefficient
ϕ	numerical function (Chap. V)
ϕ_A	d'Aubuisson numerical function
ϕ_L	numerical coefficient for larger bed forms
ϕ_N	Nagler numerical function
ϕ_S	numerical coefficient for smaller bed forms
ϕ_Y	Yarnell numerical function
ω	experimental equation exponent
ω'	modified experimental equation exponent

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The unique geographic location of Florida makes it subject to the forces of hurricanes. Excessive rainfall and abnormally high, wind-induced tides may cause extensive flooding in some coastal areas. The extent of flooding depends to a large degree on topography and the resistance to flow encountered by advancing flood waters. In hydraulic computations flow resistance is expressed in terms of friction factors. Two principal friction factors required for an analysis of coastal flooding were examined through the use of laboratory experiments. Equations and methods to determine friction factors for bed forms and for vertical obstructions were developed and verified.

Vertical obstructions are considered to include tree trunks, buildings, or other objects which extend through the water surface and create drag forces on flowing flood waters. The theoretical basis for this friction factor analysis is the drag force equation. It is shown that friction factors are proportional to drag coefficients. Several parameters of the flow and the obstructions are required, and these are specified in detail. Friction factors for flow through dense vegetation are not considered, although these factors are an important part

of an analysis of flooding. Circular cylinders were installed in a laboratory flume to simulate vertical obstructions in the coastal zone. The results of these tests were evaluated to develop equations for friction factors for vertical obstructions.

Dunes on the beach and offshore bars are similar to the bed forms which occur in inland waterways. Models of these bed forms were tested in a laboratory flume. The results of these tests were used to develop an equation from which friction factors may be determined for this type of flow resistance. This equation gives the friction factor in terms of dimensionless parameters characteristic of the flow and the bed forms. The theoretical basis for this analysis is an analogy between bed form energy losses and the energy losses due to a sudden expansion in closed-conduit flow.

CHAPTER I INTRODUCTION

Florida has hundreds of miles of coast line and has excellent weather almost year round. On both coasts much of the coastal zone has been developed regardless of the suitability for such activities. Since some natural areas remain, the coastal zone is very diverse.

Some areas of Florida's coast are characterized by extensive commercial and/or residential development with little vegetation other than that required for aesthetics. Other areas remain reasonably natural with little development. These areas may be covered by dense woods, sawgrass, mangroves, scrub, or other types of vegetation in various quantities and densities. Still other areas combine natural and developed zones in various proportions. In the event of a hurricane the combined effects of wind, rain, and tide may cause extensive flooding within the coastal zone. The lateral extent and local magnitude of such flooding depend to a large degree on topography and the resistance to flow offered by the land surface and objects on that surface. High resistance impedes advancing flood waters. This may increase local flood levels but reduces the total area effected. Low resistance tends to increase the area flooded but reduces the flood levels. For reasons of public safety and for assessment of property damage it is necessary to be able to predict the extent, magnitude, and duration of any flooding which may occur. This requires accurate estimates of the resistance to flow offered by the area in question. From a hydraulic point of view flow resistance is expressed in terms of a friction factor. The diversity of coastal areas implies that many friction factors are required if all types of areas are to be considered. The development of equations or methods by which friction factors may be determined for all possible combinations of natural and developed areas is

beyond the scope of this investigation; however, equations and methods are developed here which consider two important types of flow resistance for a coastal setting. It should be noted that these developments are not strictly limited to coastal flooding but may be applied to inland flooding as well.

Several types of flow resistance are normally considered (Christensen and Walton, 1980). It is convenient from a practical and a theoretical standpoint to arrange these various types of resistance into groups. This grouping may determine the characteristics of the area considered and/or determine the methods of analysis used. Although several groups are possible, only three groups are considered here. These groups will be called "bed forms," "vertical obstructions," and "skin friction." Only bed forms and vertical obstructions are considered in detail, and special provisions are made to determine what objects may be classified as a vertical obstruction. This classification is based on the methods of analysis used.

Friction factors are used as a measure of the resistance to flow, and high friction factors imply high resistance. In conjunction with flow parameters such as depth and discharge the Darcy-Weisbach equation is used with the friction factor to determine energy losses for a given length of flow. Energy losses are made evident by changes in the kinetic and potential energy of the flow, and in all cases the equation which expresses energy conservation must be used. This equation considers elevations, depths, velocities, length, and friction factor.

Consideration of the energy losses due to bed forms is concerned primarily with dunes in the immediate area of the beach and any bar type features which exist offshore. On a large scale these losses may be reasonably minor in comparison with the energy losses which occur farther inland. With the exception of skin friction, dunes and other bed features are the initial form of resistance encountered by a storm surge. This resistance may therefore affect the depth and velocity of the storm surge as it invades more inland areas.

Consideration of the energy losses associated with vertical obstructions is made with reference to the drag force equation. This equation will be given in Chapter II and indicates that a single object of constant cross-sectional area, shape, and orientation immersed in flowing water experiences a drag force acting on it due to the flow. When the object extends through the entire water column, the flow around the object is approximately two-dimensional. Object shapes and flow velocities may vary, but in all cases the drag force is expressed in terms of an experimentally determined drag coefficient and certain properties of the flow. Vertical obstructions in a flood situation would also experience drag forces due to the flow, and it is assumed for this analysis that the energy loss caused by a vertical obstruction is directly proportional to its drag coefficient. Since the drag force equation considers individual objects, vertical obstructions must be limited to objects which act independently or nearly so. Provisions are made for interference between objects, but the flow must pass around the object and be mainly two-dimensional. Examples of vertical obstructions would include tree trunks, buildings, power poles, or other similar objects. Objects such as shrubs, dense foliage, tall stands of grass, mangroves, or other obstructions with multiple paths of flow are not considered as vertical obstructions. The individual elements of these objects which interact with the flow do not do so independently. Vertical obstructions may be natural or man-made, and they must extend through the entire water column. They must interact with the flow in an independent manner, and the flow around them must be mainly two-dimensional. Obviously some latitude may be allowed with these criteria; however, it must be within reason.

Energy losses due to skin friction are always present in any flow, and these losses are by necessity considered in the laboratory experiments. Under normal conditions a friction factor for skin friction losses is associated with the roughness of the surface the flow passes over. Flows over paved areas, lawns, barren areas such as a beach, or other reasonably unobstructed areas would experience an energy loss associated with

the roughness of the surface, and a suitable friction factor could be determined based on this roughness or the loss of energy. This concept of roughness will be extended to include such areas as sawgrass, scrub, or other dense foliage. This may or may not be appropriate but is done to separate vertical obstruction losses from skin friction losses. These two types of loss are isolated by the equations considered in the following chapters. It is important to note a basic difference between surface roughness and friction factors. Although both may be used to determine energy losses in a particular flow, surface roughness is normally considered constant. Friction factors on the other hand are not constant but change with depth of flow. The association of a roughness with the flow of water through dense foliage probably produces a roughness which is also depth dependent.

Primary consideration is given here to friction factors for bed forms and for vertical obstructions. These friction factors may be used with the parameters of the flow to establish rates of change in depth and velocity of flood waters. Other sources of friction or energy loss are assumed to be due to skin friction. In a particular flooding situation all forms of energy loss must be considered if the analysis is to yield accurate results. The equations developed here consider bed form losses independently; however, vertical obstruction losses and skin friction losses are separated but are considered jointly in a single equation. These skin friction losses combine all forms of energy loss other than those due to bed forms and vertical obstructions.

Losses due to bed forms are examined through the use of model bed forms which are constructed and tested in a laboratory flume. Vertical obstruction losses are examined through the use of vertical sections of circular pipe in the same flume. These two forms of loss are not examined at the same time. Details of the tests and their results are given in the following chapters.

CHAPTER II

LITERATURE REVIEW

J. Nikuradse introduced the results of his, now famous, sand-roughened pipe experiments in 1933. Those results provided coordination of what had been a loosely organized body of experimental data (Nikuradse, 1933). Since then, numerous field and laboratory tests have been conducted seeking to provide an even better understanding of the energy transfers associated with the flow of water and other fluids in both open and closed conduits.

From a hydraulic point of view the closed-conduit, turbulent flow considered by Nikuradse can be classified as smooth-range flow, rough-range flow, or flow in a transition-range between smooth and rough. The rough-range experiments introduced the concept of equivalent-sand-roughness or simply roughness. Energy losses associated with this roughness are called skin-friction losses. The losses in smooth-range flow may also be called skin-friction losses, although they are not dependent on the roughness of the flow surface but are functions of the fluid's viscosity. For flow in any range it is customary to associate a friction factor with the energy losses. Friction factors can be determined for Nikuradse's experiments based on the fluid properties and/or the surface roughness.

The flow of water in a coastal flooding situation involves much more than the skin friction losses considered by Nikuradse. Horizontal obstructions such as dunes or bed forms create wakes in the flow which dissipate energy. Vertical obstructions in any form also produce wakes. Much like the friction factors for skin friction losses it is customary to use friction factors for other forms of energy loss as well. In particular, bed forms and

vertical obstructions are each assumed to have a friction factor. It is further assumed that these friction factors may be used with the Darcy-Weisbach equation to determine the energy losses. The Darcy-Weisbach equation for flow in a circular pipe is given as Equation 2-1 (Streeter, 1971). This equation will be expressed in other forms in the following chapters, and more appropriate variables will be used.

$$\Delta H = f \frac{v_m^2 L}{2g D} \quad 2-1$$

where	ΔH	=	energy loss
	f	=	friction factor
	v_m	=	mean velocity
	g	=	acceleration due to gravity
	L	=	length of pipe considered
	D	=	pipe diameter

The Darcy-Weisbach equation may be introduced into the energy equation. This equation is given for steady flow in an open channel as Equation 2-2. Subscripts 1 and 2 imply an upstream cross-section 1 and a downstream cross-section 2, respectively (Lecture Notes, CWR 4202). A typical longitudinal section of an open channel is given as Figure 2-1.

$$z_1 + d_1 \cos \beta + \alpha_1 \frac{v_{m1}^2}{2g} = z_2 + d_2 \cos \beta + \alpha_2 \frac{v_{m2}^2}{2g} + \Delta H \quad 2-2$$

where	z	=	bed elevation at point of maximum depth
	d	=	maximum flow depth
	β	=	inclination of bed with horizontal
	α	=	energy coefficient

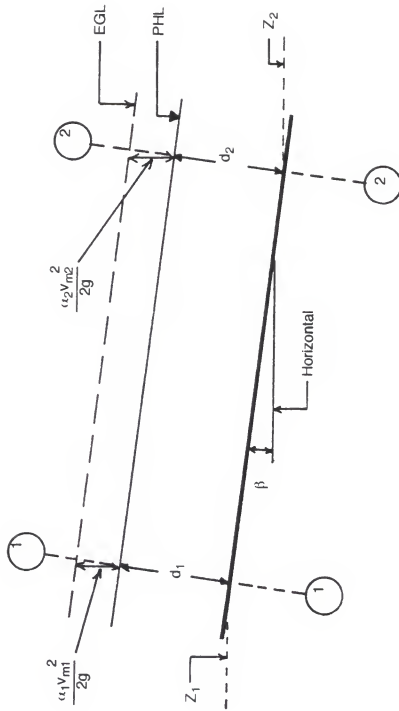


Figure 2-1: Longitudinal Section in an Open Channel

For normal conditions the bed inclination is generally very small and $\cos \beta$ may be replaced by 1.0. Note also that the Darcy-Weisbach equation in a more suitable form should actually be used in open channel flow rather than Equation 2-1 which is restricted to circular pipes.

When more than one source of energy loss is considered in a particular flow situation, it is assumed that the individual friction factors may be added to give a combined friction factor. Alam and Kennedy (1969) added the friction factors for bed forms and skin friction. Einstein and Banks (1950) conducted a series of flume experiments with different arrangements of blocks and pegs which effectively considered skin friction, bed forms, and vertical forms. They determined that the total resistance force was equal to the sum of the individual resistance forces. In effect, their friction factors were additive. Other examples of additive friction factors could also be given; however, blind addition of friction factors is not recommended. Some discretion should be used. As an example the friction factor for skin friction is modified in the current analysis. This is given in Chapter V.

Bed Forms

A review of the engineering literature reveals that the resistance to flow in erodible-bed channels is usually difficult but not impossible to determine. The properties of the flow affect the properties of the bed, and the bed, likewise, affects the flow (Graf, 1971). For low discharge, subcritical flows the channel bed may be reasonably flat such that skin friction provides the majority of the flow resistance; thus the Reynolds number and characteristics of the bed material become the major determining factors. For other low discharge, subcritical flows ripples and/or dunes may exist such that skin friction losses are of secondary importance compared to the losses associated with the zones of flow separation which occur on the lee sides of these larger bed features. For higher discharge, subcritical flows, the ripple or dune bed forms may be washed out and plane bed conditions exist. This high discharge, plane bed condition usually exhibits high rates of sediment transport. For even higher discharges the flow may become supercritical and the

bed may be plane, or form antidunes, or produce bed features causing mixed flow. An excellent outline of these various stages of flow and bed forms is provided by Simons and Richardson (1966).

As a basis for analysis the current investigation will be concerned primarily with flows which are subcritical; that is, flows for which the Froude number is less than 1.0. The Froude number is defined by the following Equation 2-3.

$$Fr_m = \frac{v_m}{\sqrt{g d_m}} \quad 2-3$$

where Fr_m = Froude number based on mean depth d_m
 d_m = mean depth defined as cross-sectional area A divided by water surface width B.

For subcritical flow conditions, an alluvial channel bed may be smooth, with or without sediment transport, or may be characterized by ripples, or dunes, or some combination thereof.

Lovera and Kennedy (1969) analyzed an extensive amount of laboratory and field data dealing with friction factors for flat-bed flows in sand channels. Their analysis considered the Prandtl-Nikuradse smooth boundary relation. This relation was used in the initial analysis of the current data. They concluded that the friction factor for such flat-bed flows depends on the Reynolds number and the grain size. They presented their results in the form of a graph giving the friction factor as a function of Reynolds number $Re = v_m R / \nu$ and the ratio R/d_{50} . Here R = hydraulic radius, ν = kinematic viscosity, and d_{50} = median grain size diameter, i.e., the grain size for which 50% by weight of the sediment particles are finer.

Hill, Srinivasan, and Unny (1969) conducted a series of laboratory tests dealing with the instability of flat beds in alluvial channels. Specifically they considered high discharge flat bed channels with suspended sediment loads and then gradually reduced the

discharge rate until ripples or dunes were formed. Their findings were presented in graphical form and defined regions of flat bed, dunes, or ripples as a function of two dimensionless variables. These variables depend on the kinematic viscosity of the fluid, the acceleration due to gravity, the median sediment particle diameter, and the friction velocity. It is assumed that this relationship could be used to determine when existing dunes and ripples are washed out to form a flat bed.

Raudkivi (1963) has examined the formation of ripples from the other extreme. Beginning with an initially flat bed with no suspended sediment in the flow, discharges were gradually increased until bed forms were developed. The formation of the rippled bed form produced marked increases in the resistance to flow. He notes that the ratio of rippled bed to flat bed shear stress, and thus the energy gradient, was approximately two. The formation of ripples doubled the friction factor. Raudkivi also associated the presence of a rippled or duned bed with the transition from hydraulically smooth to hydraulically rough flow. Consider a gradual increase in discharge from a flat bed with no sediment motion through the formation of ripples and/or dunes to the point where the bed forms are washed out and high sediment transport rates exist. Raudkivi notes that the friction factor for the latter condition is nearly as low as the friction factor at the beginning of sediment motion. Between the two extremes significantly higher friction factors were noted.

Concerning the wake on the lee side of the bed forms, Raudkivi states,

the wake flow is mainly maintained by the turbulent shear stress established in the interface region. This turbulent shear stress appears to be insensitive to the step or ripple height and the nature of the boundary layer of the approaching flow. The magnitude of the flat boundary shear stress appears to be a factor in determining the height of the ripple for any given grain size, and the amount of turbulence produced at the interface. The form drag in turn is a measure of the flow energy converted into energy of turbulence. Each ripple can be considered as an isolated disturbance, each wake having been dissipated before the next ripple crest is reached. (Raudkivi, 1963, p 31)

It is doubtful that ripples and dunes both satisfy the above description; however, some similarities should be present.

Engel (1981) conducted a series of laboratory tests with triangular roughness elements simulating dunes to investigate the length of the separation zone on the lee side of a dune crest. He concluded that the length of the flow separation behind a dune is independent of the Froude number and virtually independent of the flow depth when the Froude number is less than 0.5 and the depth ratio is greater than about 5. The depth ratio is defined as the average depth of flow divided by the dune height. He also concluded that the length of the separation zone depends on the sand grain roughness when the dunes are flat and is virtually independent of the sand grain roughness when the dunes are steep. A dune is defined as steep when its height to length ratio exceeds 0.05. Steep dunes have a flow separation length approximately four times the dune height. Engel's results for determination of the flow separation length for flat dunes were presented graphically as a function of d_{50} and the height and length of the bed form.

Engelund (1966) considered bed forms for particular flow conditions and presented results as a function of the Froude number and the ratio of mean velocity to friction velocity. He suggested the use of the Carnot equation to evaluate the expansion losses due to the bed forms. He also suggested addition of the losses due to bed forms and skin friction. His report considered the similarity of parameters for alluvial streams in general.

Rifai and Smith (1971) performed a series of experiments with triangular elements simulating dunes to examine the characteristics of the turbulence produced. Their elements were approximately 5 cm in height and 50 cm in length with an angle of about 7 degrees for the upstream face. Note that this height is roughly half the height of the larger elements used in the current analysis. They concluded that flow over the triangular elements reproduced in idealized form a number of the characteristics of flow over dune beds. They also concluded that the macroscale of the turbulence was related to the height of the roughness elements and not to the depth of flow as in a plane bed channel.

Wang and Shen (1980) conducted a series of alluvial flume experiments to investigate the statistical properties of alluvial bed forms. Their primary interests were the

relationships between dune height, length, and angle of the upstream face as a function of the flow's friction velocity. For a flow depth maintained at approximately 1.0 ft they noted a maximum angle of about 11 degrees for the upstream face and a maximum dune height of 0.4 ft. The corresponding dune height to depth ratio was 0.34. No maximum length of dune was established for the flow conditions they considered; however, dune height and length were correlated reasonably well up to the point of maximum dune height. Form friction factors were also given for twenty-two data groups. They noted a linear correlation between the friction factor and the ratio of average dune height to water depth. A portion of the data presented by Wang and Shen is tabulated in Chapter V and used for comparison with data from the current investigation.

Shen, Fehlman, and Mendoza (1980) have conducted experiments similar to the current investigation. They used triangular elements approximately 14 cm in height and 92 cm in length with an angle of 11.5 degrees for the upstream face. They used both smooth and roughened surfaces during different portions of their tests. A portion of their data is also tabulated in Chapter V and used for comparison.

Other investigators have examined the flow over bed forms in alluvial channels but have considered factors not considered in the present analysis. Their results are therefore not directly comparable with the current results.

The alluvial bed experiments conducted by Wang and Shen gave a maximum of about 11 degrees for the angle of the upstream face. Shen, Fehlman, and Mendoza used an angle of about 11.5 degrees for the upstream face. The current experiments use triangular elements with angles of about 11.2 degrees for the upstream face and 30 degrees for the downstream face. The dimensions of these elements are given in Chapter IV.

Vertical Obstructions

Over the past sixty to seventy years an extensive amount of both experimental and theoretical research has considered the flow of fluids past solid bodies. Air and water have

been the fluids of primary concern and have roughly divided the research into two categories.

Investigations involving the flow of air past solid bodies have basically sought to meet the needs of the aviation industry by providing lift and/or drag forces acting on objects and how these forces are affected by such factors as shape and flow variables. Air flow investigations have also led to the development of boundary layer theory which seeks to explain how and why the objects and fluid behave as they do. This theory is not limited to air flow and plays a significant role in the development of modern aircraft as well as automobile and marine design.

Of importance here is the interaction between flowing water and solid bodies. Solid bodies in this case implies natural or man-made obstructions which generally protrude through the water surface and which the flow must pass around. Tree trunks or buildings are examples. Objects such as lawn grass, small vegetation, curbs, or other generally small submerged items are, of course, obstructions to flow but are best handled as roughness elements for skin friction losses. In this analysis larger vegetation such as sawgrass, scrub, or dense foliage will also be included with skin friction losses.

Airfoils are probably the most thoroughly investigated solid body. Others include flat plates, spheres, discs, and cylinders of various cross-sectional shapes. One of the most important of these, at least for the current analysis, is the circular cylinder placed at right angles to the flow. Laboratory experiments using circular cylinders immersed in flowing water form the basis for the current analysis of vertical obstructions. Equations are developed in the following chapters to provide friction factors for vertical obstructions as a function of the flow and obstruction parameters. These equations are based on laboratory results, information available in engineering literature, and the drag force equation. The drag force equation and its coefficients are briefly examined in the following paragraphs.

The general form for the drag force equation was developed by Newton (Rouse, 1946; Prandtl and Tietjens, 1934). In modern form this equation is given as Equation 2-4.

$$F_D = C_D A \frac{\rho v^2}{2} \quad 2-4$$

where	F_D	=	drag force on object
	C_D	=	drag coefficient
	A	=	projected area of object in the direction of flow
	ρ	=	fluid density
	v	=	velocity of object relative to the fluid or velocity of fluid relative to the object

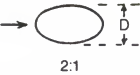
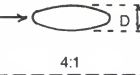
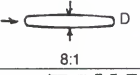
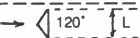
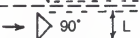
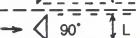
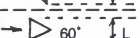
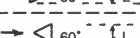
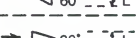
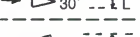
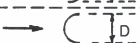
Drag coefficients have been determined experimentally by other investigations for a wide range of objects. Table 2-1 lists some of the more common objects considered in those investigations and gives the drag coefficients for two-dimensional flow around the objects (Streeter, 1971). Figure 2-2 is obtained from Giles (1962). The figure plots drag coefficients as a function of the Reynolds number for several types of objects. Note that both two and three-dimensional flows are considered. Using the current notation, the Reynolds number is defined by Equation 2-5.

$$Re_\delta = \frac{v_m \delta}{\nu} \quad 2-5$$

where	Re_δ	=	Reynolds number based on characteristic dimension δ
	δ	=	characteristic dimension
	ν	=	kinematic viscosity of fluid

The characteristic dimension has units of length and is defined by the flow being considered. For flow around obstructions the characteristic dimension is generally the width of the obstructions area projected in the direction of flow. For a circular pipe the projected area would have a width equal to the diameter of the pipe.

Table 2-1. Drag Coefficients for Two-Dimensional Flow

Shape	Flow Direction	Re	Characteristic Dimension	C_D
Circular Cylinder		$10^4 \rightarrow 1.5 \cdot 10^5$	D	1.2
Elliptical Cylinder		$4 \cdot 10^4$	D	.06
		10^5	D	0.46
		$2.5 \cdot 10^4 \rightarrow 10^5$	D	0.32
		$2.5 \cdot 10^4$	D	0.29
Square Cylinder		$3.5 \cdot 10^4$	L	$2.0 \rightarrow 2.05$
		$10^4 \rightarrow 10^5$	L	1.6
Triangular Cylinder		10^4	L	2.0
		10^4	L	1.72
		10^4	L	2.15
		10^4	L	1.60
		10^4	L	2.20
		10^4	L	1.39
		10^5	L	1.8
		10^5	L	1.0
Hollow Hemisphere		$4 \cdot 10^4$	D	2.3
		$4 \cdot 10^4$	D	1.12

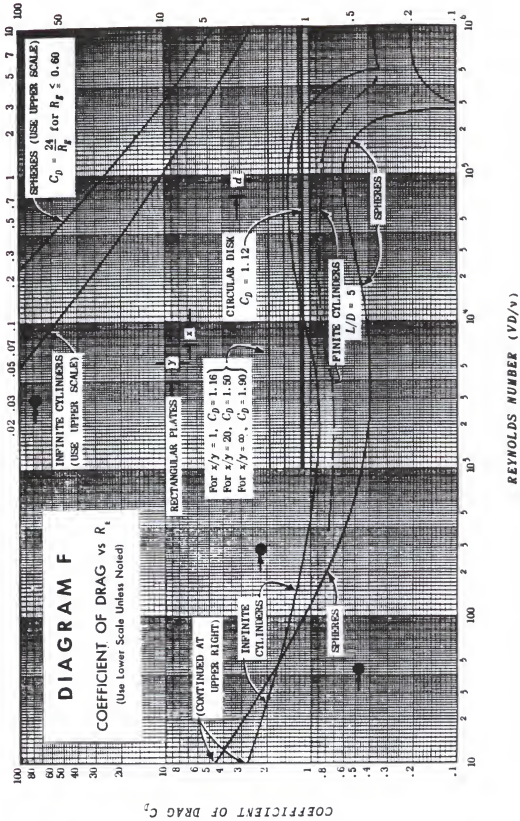


Figure 2-2: Drag Coefficients for Selected Objects

The Reynolds number may also be defined for flow in a closed conduit or in an open channel. For these flows the characteristic dimension is generally set equal to the hydraulic radius of the flow. The Reynolds number is then denoted Re and may have a subscript. Hydraulic radius is defined by Equation 2-6.

$$R = \frac{A}{P} \quad 2-6$$

where

R	=	hydraulic radius
A	=	cross-sectional area of the flow
P	=	wetted perimeter of the flow exclusive of a free water surface

The choice of circular cylinders for the current laboratory experiments is based on the accuracy of the drag coefficients for a large range of Reynolds numbers. The need for accurate drag coefficients will become apparent later. Based on Figure 2-2, a portion of the curve for two-dimensional flow around an infinite circular cylinder will be expressed by three equations. Each equation is applicable only to a limited range of Reynolds numbers. These equations and the applicable ranges are given as Equations 2-7, 2-8, and 2-9. Drag coefficients calculated through the use of these equations are given in tables in Chapter V. Equations 2-7 through 2-9 are intended for laboratory use but may be used in the field if the obstruction is a circular cylinder and the Reynolds number is within the range indicated.

$$C_D = 0.233 \ln Re_b - 1.057 \quad 2-7$$

for $5,000 < Re_b < 10,000$

$$C_D = 0.209 \ln Re_b - 0.831 \quad 2-8$$

for $10,000 < Re_b < 40,000$

$$C_D = 0.047 \ln Re_b + 0.887 \quad 2-9$$

for $40,000 < Re_b < 60,000$

where $\ln$ = natural logarithm

A detailed analysis of the curve in Figure 2-2 for the infinite circular cylinder is not necessary here, but some remarks are in order. The total drag force is made up of a viscous drag and a pressure drag. The viscous drag or skin friction drag is developed in the boundary layer which forms at the leading edge of the object and continues around the object until the layer separates. The point of separation varies with such factors as surface roughness and velocity and is very instrumental in establishing the pressure drag. Beyond the point of separation a wake forms behind the object. The pressure in the wake region is lower than the pressure on the upstream side; thus there is a net force acting on the object in the downstream direction. Only for small Reynolds numbers is the viscous drag of any consequence. For a circular cylinder and a Reynolds number as small as 200, pressure drag accounts for about 90 percent of the total drag force (Massey, 1979). A sharp drop in the drag coefficient is noted for a Reynolds number between 10^5 and 10^6 . This drop occurs because the boundary layer becomes turbulent and the point of separation moves farther around the object. The low pressure area becomes smaller and thus the drag force is decreased. A particularly thorough description of the curve in Figure 2-2 is provided by Massey (1979).

The cross-sectional shapes for several objects are shown in Table 2-1, and the direction of flow is indicated. It should be noted that the drag coefficient for some objects is not a strong function of the Reynolds number as it is with the circular cylinder. For the circular cylinder the point of separation of the boundary layer is variable and thus the drag force is variable. An object such as a cylinder which is square in cross-section does not show variation in the point of separation. Separation occurs at the upstream corners, and the drag coefficient remains relatively constant for a wide range of Reynolds numbers. There are of course limits to this relative constancy. For very small Reynolds numbers

viscous forces become important, and for very large Reynolds numbers the flow may radically separate from the surface of the object. Normal field conditions are generally well within these two extremes.

Table 2-1 and Figure 2-2 consider drag coefficients for various objects. They do not, however, consider energy losses or friction factors for these objects. Drag coefficients in general depend on shape and velocity and provide a measure of an object's capability to produce pressure variations over its surface area. The net force acting on the object is obtained by integration of the pressure over the total surface area exposed to the flow. In contrast to pressure variations and net force which consider an object's surface, energy losses for an object consider a region of flow extending outward from the surface of the object to include all portions of the flow for which shear stresses are developed due to the presence of the object. Most notable is the turbulent wake region downstream of the object.

In both open and closed conduit flow, energy loss per unit weight of fluid is measured directly by loss of pressure head when velocity heads and elevations are accounted for. Since the drag coefficient provides a measure of an object's capability to produce pressure variations and energy losses are in effect measured by losses in pressure head, there must be a proportionality between drag coefficients and energy dissipation. The experimental results presented later verify this proportionality and show the dependence of energy losses on obstruction parameters and properties of the flow.

In spite of the large amount of research which has been done in the past, only a small portion has attempted to provide friction factors for vertical obstructions as is considered here. One exception is an investigation by Wang and Christensen (1986). In their investigation drag coefficients for storm induced flows in residential areas as well as medium and high rise commercial areas are developed. These drag coefficients are used to determine a friction factor which is in turn used with the Darcy-Weisbach equation to

establish energy losses. Their work is similar in purpose to the current investigation and will be considered in more detail in Chapter V.

Other investigations have been concerned with energy losses in the flow around obstructions such as bridge piers. Some of the most notable of these are experiments done by Nagler (1918), by Yarnell (1934), and by d'Aubuisson (see Chow, 1959). The results of these experiments compare favorably with the current experimental results. Energy losses in the flow around bridge piers were determined in those investigations through the use of experimentally determined coefficients and empirical equations. These equations will be expressed in terms of the factors used for the current investigation. These factors are developed in the following chapters, and the results mentioned above will be considered in Chapter V.

Most of the investigations involving friction factors and flow obstructions have basically incorporated the obstruction losses with the skin friction losses. Examples of this include the flow of water in vegetation lined channels, sheet flow in wetlands, and various overland flow situations. These investigations do not single out individual obstructions but base the analysis on average properties of the entire obstruction system. The analysis here is concerned with obstructions to flow which may be treated as individual elements. The following chapters consider the analysis of these elements.

CHAPTER III

DEVELOPMENT OF THEORETICAL EQUATIONS

Figures which detail the boundaries of a flow region may be drawn for almost any conditions of fluid flow, whether this flow is in a open channel or in a closed conduit. Lines may be drawn in these figures to indicate the motion of the fluid. Path lines indicate the paths taken by particular fluid particles. Streamlines indicate the direction of motion at any point in the fluid at a specific time. When the properties of the flow do not change with time, the flow is called steady and path lines coincide with streamlines (Batchelor, 1958). Rapid changes in the alignment of the flow boundaries will generally cause the flow and thus the streamlines to separate from the boundary. Downstream of the point of separation wakes are formed which dissipate energy due to turbulence. Wakes are produced in the flow past vertical obstructions and in the flow over bed forms. Energy losses due to these wakes may be determined on an individual basis by experiment; however, this investigation seeks to examine these losses in a theoretical manner and include a broad range of conditions. This theoretical development is considered in this chapter.

Bed Forms

Although the geometries are different, bed forms and a sudden expansion in a closed conduit are considered analogous. The flow boundaries may be described in geometric terms, and both produce wakes which dissipate energy. Figure 3-1 depicts a longitudinal section of a sudden expansion in a horizontal, circular pipe. The upstream pipe is smaller in diameter than the downstream pipe, but both pipes share a common centerline. Cross-sections 1 and 2, respectively, indicate the position of the expansion and the point at which the streamlines reattach to the flow boundary. Approximate streamlines are

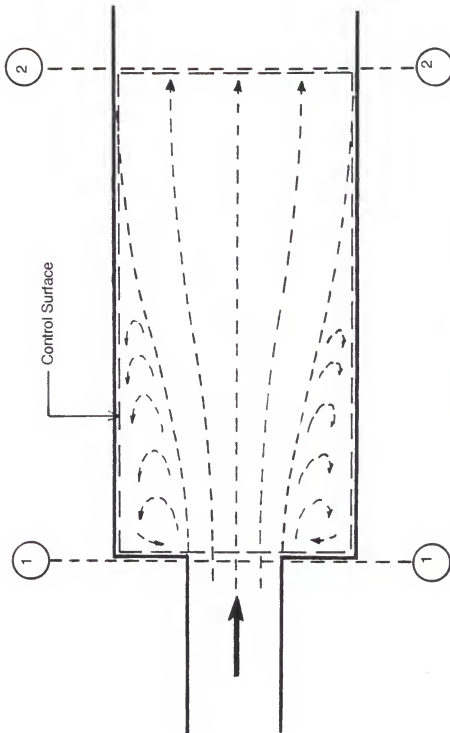


Figure 3-1: Longitudinal Section of a Sudden Expansion in a Horizontal, Circular Pipe

drawn in Figure 3-1, and the direction of flow and wake region are indicated. Equations are developed in the following paragraphs based on Figure 3-1. Subscripts 1 and 2 are used in these equations and refer to cross-sections 1 and 2, respectively, of Figure 3-1.

Application of the steady flow energy equation between sections 1 and 2 of Figure 3-1 leads to the following Equation 3-1 (Lecture Notes, CWR 4202).

$$\frac{p_{o1}}{\gamma} + \alpha_1 \frac{v_{m1}^2}{2g} = \frac{p_{o2}}{\gamma} + \alpha_2 \frac{v_{m2}^2}{2g} + \Delta H_{SE} \quad 3-1$$

where p_o = pressure at the centroid
 γ = unit weight of water
 ΔH_{SE} = sudden expansion energy loss

The steady flow momentum equation may be applied to the control surface sketched in Figure 3-1. Neglecting shear stresses on the inner walls of the conduit, the following Equation 3-2 is obtained.

$$\alpha_2' \rho \frac{Q^2}{A_2} - \alpha_1' \rho \frac{Q^2}{A_1} = p_{o1} A_2 - p_{o2} A_2 \quad 3-2$$

where α' = momentum coefficient
 Q = discharge

Note that the terms on the right-hand side of Equation 3-2 both contain the same area A_2 . The pressure at the centroid of the upstream end of the larger pipe is p_{o1} . Fluid enters the larger pipe only in the smaller area defined by the upstream pipe; however, the pressure acts over the entire area at this entrance.

The continuity equation for incompressible flow may also be applied to Figure 3-1 and leads to the following Equation 3-3.

$$v_{m1} A_1 = v_{m2} A_2$$

3-3

Combining Equations 3-1, 3-2, and 3-3 leads to the following Equations 3-4 and 3-5. These equations give the energy loss for the sudden expansion in terms of the upstream velocity head, the cross-sectional areas, and the energy and momentum coefficients.

$$\Delta H_{SE} = \zeta_{SE} \frac{v_{m1}^2}{2g} \quad 3-4$$

$$\zeta_{SE} = \alpha_1 - 2 \alpha_1' \frac{A_1}{A_2} + (2 \alpha_2' - \alpha_2) \frac{A_1^2}{A_2^2} \quad 3-5$$

where ζ_{SE} = sudden expansion energy loss coefficient

$$\frac{v_{m1}^2}{2g} = \text{velocity head}$$

The momentum coefficients which appear in Equation 3-5 may be approximated in terms of the energy coefficients. This approximation is given as Equation 3-6 (Lecture Notes, CWR 4202).

$$\alpha' = 1 + \frac{\alpha - 1}{3} \quad 3-6$$

Substitution of Equation 3-6 into Equation 3-5 gives the expansion loss coefficient in terms of the energy coefficients and cross-sectional areas. By setting the energy coefficients equal to 1.0, Equation 3-5 becomes a simple function of the cross-sectional areas. This function is the Carnot equation, and the use of this equation in the analysis of bed forms has been considered in other experiments (Engelund and Hansen, 1966). Energy coefficients are always greater than 1.0; however, they are generally close to 1.0

in turbulent flow. Typical energy coefficient values for the current experiments are on the order of 1.06 to 1.12. Direct use of the Carnot equation will be considered in Chapter V.

Figure 3-2 shows a longitudinal section of an idealized ripple or dune bed form in an open channel for which the flow is subcritical. The water surface and bed are somewhat distorted to more clearly indicate the variables considered. The figure depicts the region of flow between the crests of two, adjacent bed forms, and it is assumed that wake regions begin at or very near these crests. The distance between the crests is denoted L_b as shown, and a dotted streamline is used in Figure 3-2 to separate the main flow region from the wake. The wake shown in Figure 3-2 occurs on the lee side of the upstream bed form, and its position as shown is approximate.

Cross-sections 1, 2, and 3 are normal to the dot-dash line shown in Figure 3-2. This line represents one-half the bed form height and is not necessarily horizontal. The channel as a whole may have some bed slope. Cross-sections 1 and 3 coincide with the crests of the two, adjacent bed forms. Cross-section 2 coincides with the point of maximum surface elevation. This point of maximum surface elevation occurs over the upstream face of the bed form (Raudkivi, 1963). Measured normal to the dot-dash line, the distance from the line to the water surface at the point of maximum surface elevation is denoted d in Figure 3-2. Also measured normal to the line, the maximum distance between the line and the bed is denoted a .

The water surface depression is denoted ϵ as shown at cross-section 1. In subcritical flow the water surface and the bed are out of phase. Since the water surface depression is generally small, the distance d will be called the local-average depth of flow.

As shown in Figure 3-2 the streamline separating the wake region from the main flow reattaches to the bed in cross-section 2. In reality the point of reattachment is not fixed in position but oscillates up and down the upstream face of the bed form (Engel, 1981). It will be assumed that on average the point of reattachment for the dotted streamline

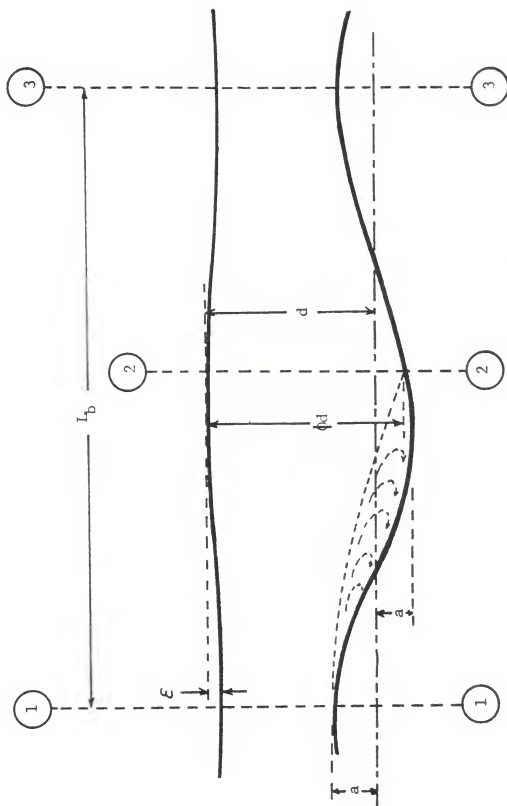


Figure 3-2: Longitudinal Section of Dune Bed Form

coincides with the point of maximum surface elevation. This assumption is consistent with the phase relationship between the bed and water surface.

Cross-section 2 does not necessarily coincide with the point of intersection of the bed and the dot-dash line. This is indicated in Figure 3-2. The depth in cross-section 2 is therefore not necessarily equal to the local-average depth of flow. The depth of flow in cross-section 2 is denoted ϕd where ϕ is a dimensionless, numerical coefficient. In general ϕ will be greater than 1.0 but less than some maximum defined by the geometry of the bed. In the form of an inequality the value of ϕ is considered to fall within the limits specified by the following Equation 3-7.

$$1 < \phi < 1 + (a/d) \quad 3-7$$

where ϕ = dimensionless numerical coefficient
 a = one-half the bed form height
 d = local-average depth of flow

As depicted in Figure 3-2 the flow region expands as the water travels from section 1 to section 2. A contraction occurs between sections 2 and 3. Equations 3-4 and 3-5 were derived for a sudden expansion in a circular pipe. By analogy these equations will be applied to the flow over bed forms. Neglecting skin friction losses and any losses which occur in the region of contracting flow, the loss of energy over length L_b of the bed forms is the expansion loss. For a channel which is rectangular in cross-section the following Equation 3-8 may be derived. This equation is based on Equations 3-4 and 3-5 and the geometry of the bed forms. The continuity equation has been used to express the mean velocity in section 1 in terms of the local-average mean velocity. The local-average mean velocity is based on the local-average depth of flow.

$$\Delta H_{1-3} = \left\{ \alpha_1 - 2\alpha_1' \frac{d-\varepsilon-a}{\phi d} + (2\alpha_2' - \alpha_2) \frac{(d-\varepsilon-a)^2}{(\phi d)^2} \right\} \frac{v_m^2}{2g} \frac{d^2}{(d-\varepsilon-a)^2} \quad 3-8$$

where	ΔH_{1-3}	=	energy loss over length L_b of the bed form
	ε	=	water surface depression
	v_m	=	$\frac{Q}{Bd}$ = local-average mean velocity
	B	=	width of flow region

The Darcy-Weisbach equation (Streeter, 1971) may be applied to closed conduit flow or to flow in an open channel. In a form suitable for open channel flow this equation is given as the following Equation 3-9.

$$\Delta H_s = f'_s \frac{v_m^2}{2g} \frac{L}{R} \quad 3-9$$

where	ΔH_s	=	energy loss due to skin friction
	f'_s	=	dimensionless friction factor for skin friction based on hydraulic radius
	L	=	length of flow region

Note specifically that Equation 3-9 considers only skin friction losses. This is indicated by the subscript s on the energy loss and friction factor terms. Numerous experiments have shown that the relationship among the variables in Equation 3-9 is indeed correct, and friction factors have been determined for many conditions of flow. Considering both laminar and turbulent flow, friction factors may depend on the roughness of the flow surface, the properties of the fluid, the geometry of the flow region, or various combinations of these parameters. The experiments performed by Nikuradse in 1933 are a classic example in the development of friction factors (Nikuradse, 1933). It is assumed that the basic form of the Darcy-Weisbach equation is correct and that energy losses may be determined by this equation for sources of loss other than skin friction. The friction factor used must of course be appropriate for the source of energy loss considered. Assuming that a suitable friction factor exists for the flow over bed forms, the Darcy-Weisbach equation will be expressed by the following Equation 3-10.

$$\Delta H_b = f'_b \frac{v_m^2}{2g} \frac{L}{R} \quad 3-10$$

where ΔH_b = energy loss due to bed forms
 f'_b = friction factor for bed forms

Combining Equations 3-8 and 3-10 leads to the following Equation 3-11. Note that the local-average depth of flow has been used for the hydraulic radius.

$$f'_b = \frac{d}{L_b} \left\{ \alpha_1 - 2\alpha'_1 \frac{d-\epsilon-a}{\phi d} + (2\alpha'_2 - \alpha_2) \frac{(d-\epsilon-a)^2}{(\phi d)^2} \right\} \frac{d^2}{(d-\epsilon-a)^2} \quad 3-11$$

Equation 3-11 may be simplified as indicated by the following Equation 3-12.

$$\frac{f'_b L_b}{2a} = \frac{1}{2a/d} \left(\frac{\alpha_1}{F^2} - \frac{2\alpha'_1}{\phi F} + \frac{2\alpha'_2 - \alpha_2}{\phi^2} \right) \quad 3-12$$

where

$$F = \left(1 - \frac{\epsilon}{d} - \frac{a}{d} \right)$$

L_b = bed form length

$L_b/2a$ = bed form length to height ratio

$2a/d$ = bed form height to depth ratio

The individual parameters in Equation 3-12 are as previously defined. If it is assumed that the water surface depression is negligible and that the energy coefficients are equal to 1.0, Equation 3-12 may be expressed as the following Equation 3-13.

$$\frac{f'_b L_b}{2a} = \frac{1}{2a/d} \left(\frac{1}{1 - (1/2)(2a/d)} - \frac{1}{\phi} \right)^2 \quad 3-13$$

Equation 3-13 implies that the bed form friction factor is a function of the length-to-height ratio, the height-to-depth ratio, and the parameter ϕ . As indicated in Equation 3-7 the parameter ϕ is also somewhat dependent on the height-to-depth ratio. The experimental

results for the current analysis yield an expression for the friction factor in terms of the two geometric ratios stated above. Energy coefficients, water surface depression, and the factor ϕ are not specifically considered in the expression; however, they are implicitly included.

Other assumptions may be made concerning the energy coefficients in Equation 3-12 which simplify the equation. Since these assumptions are somewhat ambiguous, they will not be given here. It is possible however to obtain an alternative expression for the water surface depression. This expression is iterative and is based on the energy equation given previously as Equation 2-2.

Applying the energy equation between cross-sections 2 and 3 of Figure 3-2 leads to the following Equation 3-14. Individual elements in this equation are as defined previously.

$$\frac{\varepsilon}{d} = \frac{1}{2} \left(\frac{Fr_m}{\phi} \right)^2 \left(\frac{\alpha_1 \phi^2}{F^2} - \alpha_2 \right) \quad 3-14$$

The Froude number in Equation 3-14 is based on the local-average mean velocity and local-average depth of flow. Froude numbers greater than 1.0 imply supercritical flow. Froude numbers less than 1.0 imply subcritical flow as considered by Figure 3-2 and the experiments.

The results of experiments are given in Chapter V and are compared with the theoretical expressions developed in this chapter.

Vertical Obstructions

When flood waters flow around vertical obstructions, wakes are produced which dissipate energy through turbulence in the flow. Theoretical consideration of these energy losses must consider not only the energy loss for a particular obstruction but a suitable method to describe the entire flow field and multiple obstructions.

In this analysis individual obstructions are considered by analogy with the drag force equation. Flowing flood waters apply a drag force on any obstruction, and the obstruction applies an equal but opposite force on the water. The force acting on the water multiplied by the velocity of the water implies a rate at which work is done on the water. Velocities in flood waters are not uniform, and the turbulence generated by an obstruction may persist in the flow for great distances from its point of origin. It is assumed that the rate of energy dissipation is proportional to the rate at which work is done on the flowing water. The parameters necessary for this proportion are given by the drag force equation, the energy equation, and the geometry of the obstructions.

Flow fields with multiple obstructions are considered through the use of an equivalent system of obstructions. The equivalent system exists in the laboratory, and the actual system exists in the field. It is necessary to distinguish between these two systems.

Field areas considered for analysis will be called "natural" areas, and a subscript n will be used to denote certain variables in the natural setting. In the laboratory an "equivalent" system is developed, and a subscript e is used for some of the variables in the laboratory or equivalent setting.

The use of the term natural is independent of whether or not the objects, i.e., vertical obstructions in the field area, are truly natural, or are man-made, or are some combination of both. Natural areas may include any reasonable combination and number of objects of various types. It will be assumed that each of these objects has a characteristic dimension δ_n which is defined by the shape of the object and its orientation in the flow field. Each object also has a drag coefficient C_{Dn} defined not only by shape but by velocity, implying both magnitude and direction. It will further be assumed that each object is vertical and has a horizontal, cross-sectional area which may be defined as $C_n \delta_n^2$ where C_n is a shape factor. Since δ_n depends on orientation of the object in the flow field, C_n must also depend on orientation if the product $C_n \delta_n^2$ is to equal the horizontal, cross-sectional area of the object.

The general form for the drag force equation was developed by Newton (Rouse, 1946; Prandtl and Tietjens, 1934). In modern form this equation is given as Equation 3-15. This equation will be used to develop a theoretical expression for a friction factor f_T . Based on experimental results this theoretical expression will be modified to obtain an actual friction factor denoted f_D which may be used with the Darcy-Weisbach equation to determine energy losses due to vertical obstructions. The subscript T implies theoretical, and the subscript D implies drag since the drag force equation is the basis for the development of the actual friction factor. The following paragraphs explain the general form for the theoretical f_T equation; however, the specific form suitable for field application will be developed in Chapter V. It will be seen that f_D depends on velocity, characteristic dimensions, drag coefficients, depth of flow, obstruction geometry, and obstruction density. Obstruction density, or more correctly, an equivalent obstruction density will be defined in the following paragraphs.

The drag force equation for a single object may be written as follows. Note that a subscript n or e is being omitted temporarily.

$$F_D = C_D \rho A \frac{v^2}{2} \quad 3-15$$

where

F_D	=	drag force
C_D	=	drag coefficient
ρ	=	fluid density
A	=	area of object projected at right angles to the flow, i.e., the frontal area
v	=	uniform, free-stream velocity

This equation may be put in a slightly different form by noting that for field or laboratory conditions the projected area A is equal to the characteristic dimension δ multiplied by the depth of flow d . Also, the velocity of the water approaching an object will

not be uniform as may be theoretically desirable. Velocity profiles taken in the laboratory flume with no obstructions in place give the typical logarithmic profile. Velocity profiles taken in the field under storm conditions should also differ from uniformity. It will therefore be assumed that the obstruction-free, mean velocity v_m is representative of the uniform, free-stream velocity v , and Equation 3-15 will be given in the following form, Equation 3-16.

$$F_D = C_{Dp} \delta d \frac{v_m^2}{2} \quad 3-16$$

where δ = characteristic dimension of object
 d = depth of flow
 v_m = obstruction-free, mean velocity

Noting that the obstruction-free, mean velocity v_m is equal to total discharge Q divided by cross-sectional area of flow Bd , Equation 3-16 may be written in the following form where B is the width of the flow field exclusive of objects and q is the discharge per unit width.

$$F_D = \frac{\rho d Q^2}{2 B^2 d^2} C_{D\delta} = \frac{\rho q^2}{2d} C_{D\delta} \quad 3-17$$

where $q = Q/B =$ discharge per unit width

The substitution of $Q/(Bd)$ in Equation 3-16 is consistent with mean velocity v_m defined as obstruction-free, mean velocity. This in turn is consistent with free-stream velocity v used for the evaluation of drag coefficients. In this analysis, v , v_m , and $Q/(Bd)$ are considered equivalent. It should be noted, however, that the actual average width of the flow field B_s will be less than B since some of the lateral area A_L is occupied by objects. To define a spatially averaged mean velocity in terms of discharge Q and

average width B_s , the following Equations 3-18 and 3-19 provide an alternative value of v_m for the entire flow field.

$$v_s = \frac{Q}{B_s d} \quad 3-18$$

where v_s = spatially averaged mean velocity
 Q = total discharge
 d = depth of flow
 B_s = actual average width of flow field

$$B_s = \left[1 - \frac{A_o}{A_L} \right] B \quad 3-19$$

where B = width of obstruction-free flow field
 A_o = total area occupied by objects
 A_L = total area of flow field and objects

Although v_m defined as $Q/(Bd)$ will be used in the development of equations, v_s is of significance since velocity effects the skin-friction shear stress acting on the flow area between objects. For this initial analysis the energy losses due to skin friction are assumed to be computed separately from the losses due to drag forces. When drag force losses and skin friction losses are considered simultaneously, the spatially averaged velocity v_s will be the most appropriate velocity for computations concerning skin friction losses. v_m will be used for drag force losses.

For a specific field or laboratory area of moderate length, the factors ρ , d , and v_m in Equation 3-16 or the factors ρ , d , and q in Equation 3-17 may be considered reasonably constant. Drag coefficients C_D will have small variation, but δ may vary considerably.

Factoring out reasonably constant terms and summing all forces F_D over area A_L , Equation 3-17 may be written in the following form, Equation 3-20.

$$\sum_N F_D = \frac{\rho q^2}{2d_m} \sum_N C_D \delta \quad 3-20$$

where N = number of obstructions in area A_L
 d_m = mean depth of flow over area A_L

Since C_D has only minor variation compared to δ , define a term λ as indicated by the following Equation 3-21.

$$\lambda = \frac{\sum_N C_D \delta}{N C_{Dm} \delta_m} \quad 3-21$$

where λ = distribution factor
 C_{Dm} = mean value of drag coefficient
 δ_m = mean value of δ

Making the substitution of λ into Equation 3-20 and dividing both sides of the equation by area A_L , the equation may be written as follows when the right hand side is multiplied and divided by δ_m .

$$\tau_D = \lambda \frac{\rho q^2}{2d_m \delta_m} C_{Dm} P_D \quad 3-22$$

where $\tau_D = \left[\frac{\sum F_D}{N} \right] / A_L =$ equivalent obstruction shear stress
 N = number of obstructions in area A_L
 F_D = drag force on one obstruction
 $\sum_N F_D$ = total drag force on all obstructions in area A_L
 A_L = gross area under consideration (obstruction area plus flow area)
 λ = distribution factor

ρ	=	fluid density
q	=	Q/B = discharge per unit width
Q	=	total discharge
B	=	width of obstruction-free flow field
d_m	=	mean depth of flow in area A_L
δ	=	characteristic dimension of one obstruction
δ_m	=	mean value of δ for all obstructions
C_D	=	drag coefficient for one obstruction
C_{Dm}	=	mean value of C_D for all obstructions
P_D	=	$(N\delta_m^2)/A_L$ = equivalent obstruction density
A_D	=	$N\delta_m^2$ = equivalent drag area

The term τ_D in Equation 3-22 is the sum of all the obstruction drag forces divided by the total area A_L over which these drag forces occur. It represents an equivalent shear stress and will be used in conjunction with the usual skin friction shear stress to obtain a theoretical friction factor f'_T for the drag forces due to vertical obstructions. The term P_D in Equation 3-22 represents an equivalent drag area A_D divided by total area A_L . This area $A_D = N\delta_m^2$, in general, will not equal the total area occupied by obstructions. In spite of this, the quotient $A_D/A_L = (N\delta_m^2)/A_L$ will be defined as equivalent obstruction density. Note that this quotient is dimensionless.

In terms of mean velocity, Equation 3-22 may be written in an alternative form given as the following Equation 3-23. This form is more convenient for the substitutions which follow.

$$\tau_D = \lambda \frac{\rho v_m^2}{2} \frac{d_m}{\delta_m} C_{Dm} P_D \quad 3-23$$

Under normal conditions of open channel flow it may be shown that the average bed shear stress τ_o is given by the following relationship, Equation 3-24. Note that this equation applies specifically for skin friction losses.

$$\tau_o = \gamma R S_{EGL} \quad 3-24$$

where τ_o = average bed shear stress
 S_{EGL} = slope of the energy grade line

The slope of the energy grade line depends on several factors as indicated in the following Equation 3-25 which is based on the Darcy-Weisbach equation.

$$S_{EGL} = f' \frac{v_m^2}{2g} \frac{L}{R} \quad 3-25$$

Combining Equations 3-24 and 3-25 gives the following expression for average bed shear stress τ_o .

$$\tau_o = f' \rho \frac{v_m^2}{2} \quad 3-26$$

Equating the expression for skin friction shear stress given by Equation 3-26 with the expression for vertical obstruction shear stress given by Equation 3-24 allows determination of the theoretical friction factor f'_T for vertical obstructions. This is given by the following Equation 3-27.

$$f'_T = \lambda \frac{d_m}{\delta_m} C_{Dm} P_D \quad 3-27$$

where f'_T = theoretical friction factor for vertical obstructions

Friction factors f'_T calculated by an equivalent form of Equation 3-27 are compared with experimentally determined friction factors in Chapter V. If all vertical obstructions are circular in cross-section and of the same diameter, Equation 3-27 may be given in the following equivalent form. Note that $\lambda = 1.0$ in this case.

$$f'_T = \frac{4}{\pi} C_D \frac{d_m}{\delta} \frac{A_o}{A_L} \quad 3-28$$

where $4/\pi$ = $1/C_e$ = reciprocal of shape factor for circular obstruction

C_D = drag coefficient for circular obstruction of diameter δ

d_m = mean depth of flow

δ = diameter of circular cylinder

A_o = total cross-sectional area of all circular cylinders

A_L = total area under consideration

Equation 3-28 may of course be developed directly from Equation 3-16 and is slightly easier to use than Equation 3-27. It specifically includes the total cross-sectional area A_o of the vertical obstructions. Equation 3-27 is a general form for the theoretical friction factor and is based on equivalent drag area A_D . Equation 3-28 is specific to equal diameter, circular obstructions and is based on actual area A_o . This form is used for calculation of f'_T in the laboratory. Note that the apparent drag area A_D is equal to (A_o/C_e) for equal diameter, circular obstructions.

Equations 3-27 and 3-28 should be expected to break down for high density situations. When the longitudinal distance between vertical obstructions is comparable to or shorter than the length of the wake, there is a sheltering effect which reduces the approach velocity and thus the drag force on the sheltered object. In addition, sheltered objects are subject to an increased amount of turbulence in the approaching flow, and this

reduces the drag coefficient. The previous equations assumed independence of the flows around objects and in the wakes behind them. Basically the drag forces were summed and by suitable manipulation were converted to a theoretical friction factor f'_T which when modified according to experiment may be used with the Darcy-Weisbach equation. Skin friction losses were assumed to be calculated independently of the drag force losses. The following analysis also sums the drag forces, but skin friction losses are considered explicitly.

Natural areas are assumed to have area A_L and it is further assumed that on this area there are obstructions to flow which have a range of δ_n values between a minimum and maximum value. If this range of values is divided into a reasonable number of intervals a histogram may be developed which associates with each interval a particular number of obstructions, i.e., a frequency distribution. Dividing the frequency of each interval or class by the total number of obstructions gives the relative frequency distribution. These distributions may be used with probability theory to calculate the mean value of δ in the preceding Equations 3-21, 3-22, 3-23 or 3-27. It was assumed for those equations that the number of obstructions was relatively small and a simple arithmetic mean was easier to compute. In the following analysis the original distribution of δ values will be modified to account for shape factors.

The natural area will be compared to an equivalent area also of size A_L but on which there are a particular number N_θ of vertical, circular obstructions all of which have the same characteristic dimension δ_θ . The number N_θ and size δ_θ of the obstructions on the equivalent area are to be selected such that the natural area and the equivalent area have the same energy loss. This requires that the depth of flow d , the mean velocity v_m , the equivalent sand roughness k , and the fluid density ρ be the same on the natural and equivalent areas, respectively. Equality of energy losses is established by setting the skin friction forces and drag forces, respectively, on the two areas equal to each other. This is considered in detail in the following development.

Equation 3-16 given previously is given again here.

$$F_D = C_D \rho \delta d \frac{v_m^2}{2} \quad 3-16$$

The total force acting on natural area A_L will be the sum of each individual drag force as indicated by Equation 3-16 plus the skin friction shear stress multiplied by the flow area within area A_L . Note that this flow area is not the same as the total spatial or lateral area A_L . This total force is given by the following Equation 3-29 for which the summation is over each obstruction in area A_L .

$$F_n = \sum_{i=1}^{N_n} \left[C_{Dn} \rho \delta_n d \frac{v_m^2}{2} \right]_i + \tau_{on} \left[A_L - \sum_{i=1}^{N_n} (C_n \delta_n^2)_i \right] \quad 3-29$$

where

F_n	=	total of drag and shear forces on natural area A_L
i	=	index of summation
N_n	=	number of obstructions in natural area A_L
C_{Dn}	=	drag coefficient for natural obstruction i
δ_n	=	characteristic dimension for natural obstruction i
τ_{on}	=	skin friction shear stress acting on flow area within natural area A_L
A_L	=	area under consideration
C_n	=	shape factor for natural obstruction i

Now consider the equivalent area also of size A_L but with N_g circular obstructions all of the same characteristic dimension δ_g . The total force acting on the equivalent area is given by Equation 3-30.

$$F_e = N_e \left[C_{De} \rho \delta_e d \frac{v_m^2}{2} \right] + \tau_{oe} [A_L - N_e C_e \delta_e^2] \quad 3-30$$

where

- F_e = total of drag and shear forces on equivalent area A_L
- N_e = number of obstructions in equivalent area A_L
- C_{De} = drag coefficient for circular obstruction of size δ_e
- δ_e = characteristic dimension for circular obstruction
- τ_{oe} = skin friction shear stress acting on flow area within equivalent area A_L
- C_e = shape factor for circular obstruction

If the natural area A_L and the equivalent area A_L are to have the same friction factor, it must be required that they have the same total force acting on them. This requires that the two areas have the same total drag force and the same force due to skin friction. The equality of drag forces is indicated in the following Equation 3-31.

$$\sum_{i=1}^{N_n} \left[C_{Dn} \rho \delta_n d \frac{v_m^2}{2} \right]_i = N_e \left[C_{De} \rho \delta_e d \frac{v_m^2}{2} \right] \quad 3-31$$

Density ρ , depth d , and mean velocity v_m , respectively, are considered equal in the natural and equivalent areas and factor out of Equation 3-31. Use will again be made of the distribution factor λ as considered earlier. When common terms are factored out, Equation 3-31 reduces to the following form.

$$\lambda N_n C_{Dnm} \delta_{nm} = N_e C_{De} \delta_e \quad 3-32$$

where

- δ_{nm} = mean value of δ_n for natural area
- C_{Dnm} = mean value of C_{Dn} for natural area

The requirement of equal forces due to skin friction is indicated by Equation 3-33 .

$$\tau_{on} \left[A_L - \sum_{i=1}^N (C_n \delta_n^2)_i \right] = \tau_{oe} (A_L - N_e C_e \delta_e^2) \quad 3-33$$

An expression for shear stress τ_o may be obtained by assuming a very wide open-channel with flow in the Manning range. This expression is given by the following Equation 3-34 which is dimensionally correct.

$$\tau_o = \frac{\rho v_m^2}{68.06} \left(\frac{k}{d} \right)^{1/3} \quad 3-34$$

where τ_o = average skin friction shear stress
 k = equivalent sand roughness

This equation is the basis for the previously mentioned equalities of factors in the natural and equivalent systems. Again, these factors are fluid density, mean velocity, equivalent sand roughness, and depth of flow. Since the total area A_L is the same in both systems and the shear stresses are set equal, Equation 3-33 reduces to the following Equation 3-35. This equation implies that the total spatial area occupied by obstructions is the same in both systems.

$$\sum_{i=1}^{N_n} (C_n \delta_n^2)_i = N_e C_e \delta_e^2 \quad 3-35$$

Defining a modified characteristic dimension δ'_n as the product $(C_n^{1/2}) \delta_n$ allows the summation in Equation 3-35 to be written as the sum of $(\delta'_n)^2$. The total number of objects has not changed, but the modified frequency distribution of δ'_n values will have, in general,

a different frequency for each interval and possibly an expanded range of intervals. Dividing the frequencies in the modified distribution by the total number of obstructions gives a relative frequency distribution for δ'_n . The relative frequency for each interval is a function of δ'_n and will be denoted $\Phi(\delta'_n)$. Equation 3-35 may now be written in the following form where both sides of the equation are divided by N_n , the number of obstructions in the natural system. Note that the summation is no longer over individual obstructions but over the modified relative frequency distribution where the modified characteristic dimensions are grouped into intervals each of which has a relative frequency.

$$\sum_{\delta'_n} \Phi(\delta'_n) (\delta'_n)^2 = \frac{N_e}{N_n} C_e \delta_e^2 \quad 3-36$$

where

$$\delta'_n = \sqrt{C_n} \delta_n = \text{modified characteristic dimension and representative of each small interval}$$

$$\Phi(\delta'_n) = \text{relative frequency for the interval specified by the } \delta'_n \text{ value}$$

Borrowing from probability theory (Benjamin and Cornell, 1970), the summation in Equation 3-36 may be replaced by an integral. This integral gives the expected value of $(\delta'_n)^2$ and is equal to the sum of the square of the mean value of δ'_n and the variance. This is given by the following Equation 3-37.

$$\sum_{\delta'_n} \Phi(\delta'_n) (\delta'_n)^2 = \int_{-\infty}^{\infty} \Phi(\delta'_n) (\delta'_n)^2 d\delta'_n = (\delta'_{nm})^2 + \sigma^2 \quad 3-37$$

where $\delta'_{nm} = \text{mean value of } \delta'_n$

$\sigma^2 = \text{variance of } \delta'_n \text{ distribution}$

Thus Equation 3-33 reduces to the following Equation 3-38 where each term has been defined.

$$(\delta'_{nm})^2 + \sigma^2 = \frac{N_\theta}{N_n} C_\theta \delta_\theta^2 \quad 3-38$$

Combining Equations 3-32 and 3-38 allows determination of the characteristic dimension δ_θ and the number of obstructions N_θ . These factors are indicated by the following Equations 3-39 and 3-40.

$$\delta_\theta = \frac{C_{D\theta}}{C_\theta} \left[\frac{\delta_{nm}'^2 + \sigma^2}{\lambda C_{Dnm} \delta_{nm}} \right] \quad 3-39$$

- where
- δ_θ = characteristic dimension of vertical, circular obstruction in equivalent system
 - $C_{D\theta}$ = drag coefficient for circular obstruction
 - C_θ = $\pi/4$ = shape factor for circular obstruction
 - δ'_{nm} = mean value of δ'_n
 - δ'_n = $(C_n^{1/2}) \delta_n$ = modified characteristic dimension in natural system
 - C_n = shape factor for an object in the natural system
 - σ^2 = variance of δ'_n distribution
 - δ_{nm} = mean value of δ_n
 - δ_n = characteristic dimension in natural system
 - C_{Dnm} = mean value of C_{Dn} in natural system
 - C_{Dn} = drag coefficient for an object in the natural system
 - λ = distribution factor

$$N_\theta = \frac{\lambda N_n C_{Dnm} \delta_{nm}}{C_{D\theta} \delta_\theta} \quad 3-40$$

- where
- N_θ = number of obstructions in equivalent system
 - N_n = number of obstructions in natural system

For purposes of review two systems have been considered—the natural system for which a friction factor is desired and the equivalent system which theoretically has the same friction factor. Before any analysis can be done certain variables must be specified. The most important of these is depth of flow. There is nothing unusual in this since even normal open-channel flow analysis requires specification of depth for at least one boundary. The specification of depth also influences how obstructions to flow are classified. For small depths a particular obstruction may protrude through the water surface and be classified as a vertical obstruction. For larger depths it may be better to consider the same obstruction as a roughness element for skin friction. The basic assumption here is that objects protrude through the water surface or have tops which are close enough to the surface to ensure primarily two-dimensional flow. When the top of an object is not close to the surface, the flow over and around the object becomes three-dimensional, and this greatly influences the drag coefficient.

In addition to depth of flow it is also necessary to specify velocity, not only in magnitude but in direction. The direction of flow determines the characteristic dimension for non-circular objects, and the magnitude of flow may influence the drag coefficient. Direction of flow may also influence the drag coefficient. It was stated earlier that the actual friction factor f'_D depends on velocity and depth of flow as well as on the obstruction system parameters. When depth and velocity are specified, the friction factor for a given system may be determined for those conditions. This in turn determines, by use of the Darcy-Weisbach equation, the loss in energy for a specified length of flow. In open-channel flow the energy losses are determined by changes in total energy which includes depth and velocity head. Thus a complete analysis of a given field area is iterative. Depths and velocities must not only satisfy the Darcy-Weisbach equation but must be consistent with the friction factor used in the equation. This friction factor is a function of depth and velocity.

Once velocity and flow depth are specified for the natural system, all of the variables in Equations 3-39 and 3-40 pertaining to the natural system may be determined. Using circular pipe, in the equivalent system fixes the shape factor C_e in Equation 3-39 at $\pi/4$. Since velocities and depths, respectively, in both systems are considered equal, the drag coefficient C_{De} in Equation 3-39 may be determined by minor iteration. C_{De} depends on δ_e for circular pipe and δ_e depends on C_{De} as indicated by the equation. Thus δ_e and N_e may be determined. This in turn fixes the total area occupied by vertical, circular cylinders in the equivalent system. This area is obtained from Equation 3-35 and is given as A_o in the following Equation 3-41.

$$A_o = C_e N_e \delta_e^2 \quad 3-41$$

The equivalent drag area in the equivalent system is equal to A_o divided by C_e ; however, the term required by Equation 3-27 is the equivalent obstruction density P_D . This is given by the following Equation 3-42 where it should be recalled that both systems have the same lateral area A_L .

$$P_D = \frac{A_o}{C_e A_L} \quad 3-42$$

Not omitting the skin friction losses, the equations developed above imply that for any natural system of vertical obstructions there is a unique number and size of vertical, circular obstructions which will produce the same energy loss as the heterogeneous mixture of elements in the natural system. Both systems are assumed to have the same depth of flow, velocity, fluid density, equivalent sand roughness, total lateral area, and total area of obstructions. Examination of Equation 3-39 reveals that δ_e is roughly equal to the mean value of δ_n . If this value is large then it is not practical to set up a laboratory experiment with this δ_e value. Thus equations for friction factors in the equivalent system

must be developed such that they apply to a wide range of variables. In part this is done by considering dimensionless quantities which are independent of the scale of the system. Not all variables involved may be treated in this manner as will be seen in Chapter V which contains laboratory results.

CHAPTER IV

LABORATORY EQUIPMENT AND TEST SETUPS

All experiments were conducted in the Hydraulic Laboratory's horizontal bed, recirculating flume. A plan drawing of the flume showing the relative position for each of its major components is provided as Figure 4-1. Figures 4-2 through 4-7 are photographs of specific pieces of equipment used during testing. These items are discussed and noted in the following paragraphs

Water was removed from the sump area by means of a low head propeller pump powered by a 75 kilowatt electric motor. Flow through the pump could be diverted back to the sump area through a by-pass valve or could pass through a flow-control valve to be circulated around the flume. By controlling the positions of the by-pass valve and flow-control valve, discharges in the flume could be varied continuously from zero up to the maximum capacity of about 40 cubic feet per second (cfs). The maximum discharge for any test was limited to about 10 cfs.

Water passing through the flow-control valve was released to a stilling-basin for which the outflow was metered by a calibrated, 90 degree, v-notch weir (Figure 4-2). The actual amount of water circulating in the flume was determined by measuring the head of the weir, i.e., the elevation of the water surface in the stilling-basin relative to the bottom of the v-notch. This elevation was determined by measuring the elevation of the water surface in a small stilling-well hydraulically connected to the stilling-basin. A calibrated point gauge was used to determine surface elevations in the stilling-well (Figure 4-3). Based on the point gauge reading, the following Equations 4-1 and 4-2 were used to calculate the head on the weir and discharge in the flume, respectively.

$$H = \frac{(P.G. - 5.62 \text{ cm})}{30.48 \text{ cm/ft}} \quad 4-1$$

where H = head on the weir in feet
 $P.G.$ = point gauge reading in centimeters
 5.62 cm = calibration coefficient for point gauge

$$Q = 2.840H^{2.514} \quad 4-2$$

where Q = flume discharge in cubic feet per second

Equations 4-1 and 4-2 were used to calculate the discharge for all laboratory tests and are independent of which test was considered. Three different sets of tests were conducted, and the laboratory setups for each of these three will be considered in this chapter. Numerical results of the experiments will be considered in the following chapters.

Before entering the test flume, flow over the weir was allowed to pass through a series of baffles and flow straighteners which reduced the amount of large-scale turbulence in the flow (Figure 4-4). Reducing this turbulence produced more uniform conditions of flow at the upstream end of the test flume.

The test flume is 8 ft in width and approximately 96 ft in length; however, the active area of flow was reduced to 4 ft in width for all tests. This reduction was made by placing a concrete block wall adjacent to the center line of the test flume for its full length. The inactive portion of the test flume was sealed from the area of flow except for small openings in the bed of the flume which allowed approximate equalization of the water surface elevations on both sides of the block wall. Sealing was accomplished through the use of a continuous strip of plastic sheeting along the block wall and by plywood sheets at the upstream and downstream ends of the test flume.

Depths of flow in the test flume could be controlled by changing the position of a vertical gate at the downstream end of the flume (Figure 4-5). Any depth between the free-flow minimum and channel-capacity maximum could be attained. Water which passed

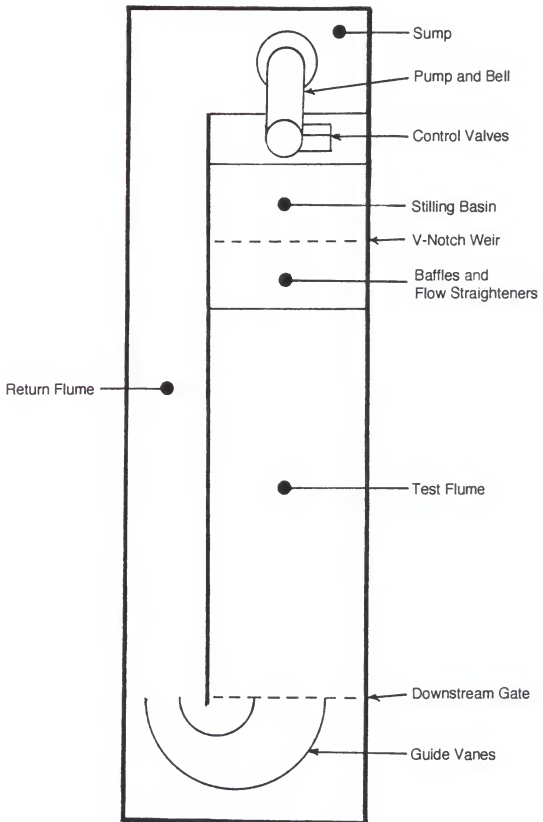


Figure 4-1: Plan View of Laboratory Flume

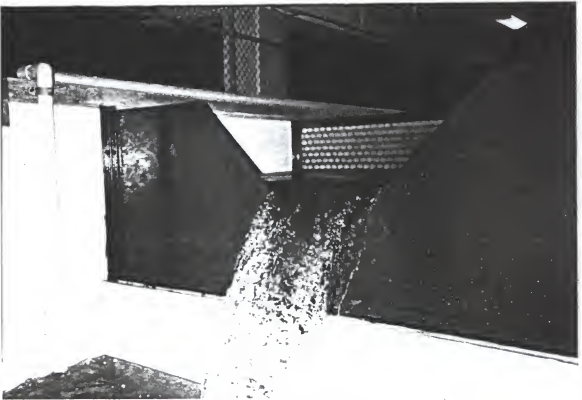


Figure 4-2: V-Notch Weir

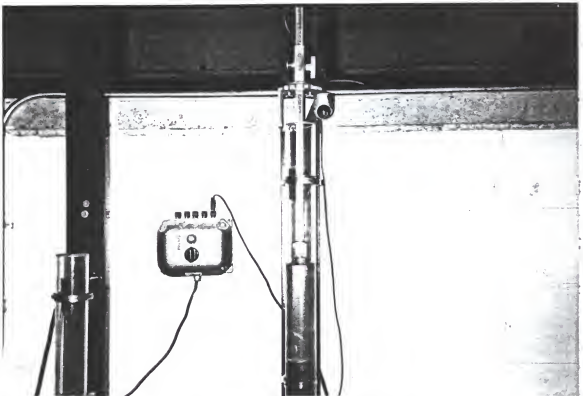


Figure 4-3: Point Gauge and Stilling Well

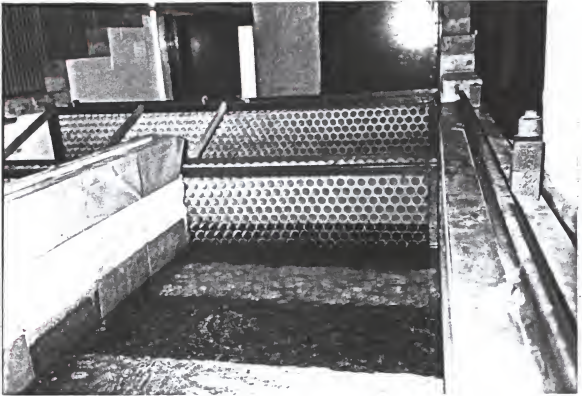


Figure 4-4: Baffles and Flow Straighteners



Figure 4-5: Downstream Vertical Gate

under the vertical gate entered the return flume and was directed back to the sump area to complete the circulation.

For all tests it was necessary to determine depths of flow at a minimum of two locations along the test flume. Depth differences and discharge could be used to determine energy losses in the flow. Up to five piezometers were placed in the bed of the test flume, and one-fourth inch inside diameter, flexible, plastic tubing was used to hydraulically connect each piezometer to a separate standpipe. All standpipes were located at a position about midlength of the test flume (Figure 4-6). The elevation of the water surface at each piezometer location was indicated by the elevation of the water surface in its standpipe. Surface elevations in the standpipes were measured using a cathetometer with a vernier scale which could be read to the nearest one-thousandth of a centimeter (Figure 4-7). This degree of accuracy does not imply that surface elevations in the flume could be defined so closely. Extreme care was taken for all readings; however, under the best of conditions small oscillations were always present in the surface elevations. The period of these oscillations varied but was on the order of 2 to 4 minutes. It is estimated that surface elevation readings are only accurate to the nearest one-hundredth of a centimeter.

Piezometers were constructed of one-eighth inch inside diameter, rigid, plastic tubing and spanned the full 4 ft of the test flume. Holes of one-sixteenth inch diameter spaced at 4 inches were drilled into one side of the tubing. These holes were placed vertically upward in all cases. One end of the piezometer tubing was closed, and the other was open and connected to the flexible plastic tubing for the standpipe. Standpipes were constructed of 2 inch inside diameter, rigid, plastic tubing.

Skin Friction Setup

Skin friction losses were reasonably small for the bed form and vertical obstruction tests; however, it was necessary to determine the magnitude of these small losses.

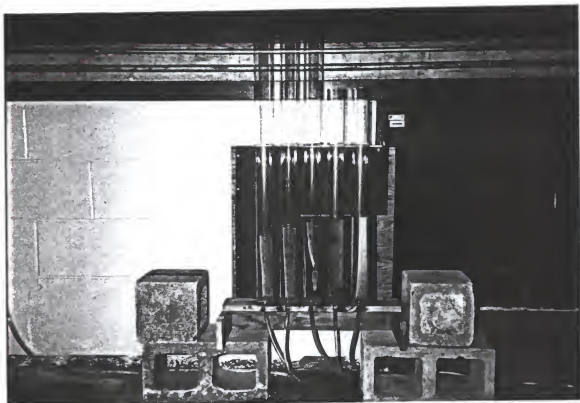


Figure 4-6: Stand Pipes

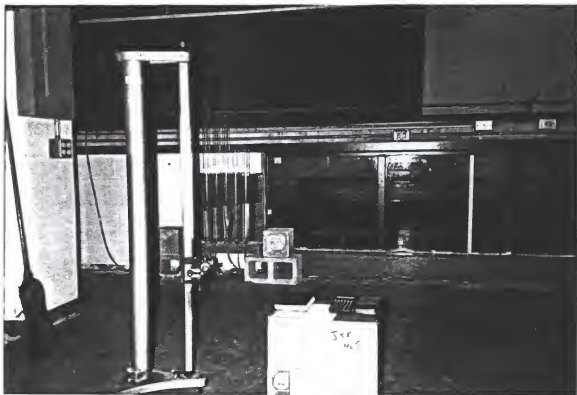


Figure 4-7: Cathetometer

The test flume was limited to a width of 4 ft and a length of about 96 ft. Over this length a smooth, galvanized sheet-metal bed was attached to the painted concrete bed usually used for tests. The galvanized sheet-metal was elevated approximately an inch above the concrete by means of a matrix of 1 inch by 4 inch boards. The metal and matrix were secured by nails, sheet-metal screws, and small concrete-anchors.

Starting at approximately 10 ft downstream of the upstream end of the sheet-metal, a series of five piezometers spaced at 19.542 ft were installed in the bed of the flume. These piezometers were countersunk into the bed with the openings in the side of the tubing at the same elevation as the surface of the sheet-metal. Silicone was used to fill any voids at the edges of the tubing and provide a smooth flow surface. This smooth, sheet-metal bed with five piezometer locations was used to determine energy losses due to skin friction for the sheet-metal. This same sheet-metal bed was subsequently used for the vertical obstruction tests. It should be noted that the bed forms were constructed of the same type of sheet-metal.

Readings of water surface elevation were recorded for all five piezometers; however, only two readings were used to determine the head losses. Based on the series of recorded values, the two values considered most accurate were selected. In some cases an average of two adjacent piezometer readings was used and the length of the flow region was adjusted accordingly. Numerical values for the piezometer readings and flow lengths are tabulated in the following chapter; however, certain relationships applicable to these tests and equipment will be given here.

Depths of flow, bed elevations, and cathetometer readings were coordinated such that flow depths could be determined directly from the cathetometer readings. This coordination was necessary since the vernier scale of the cathetometer could be set at an arbitrary elevation relative to the elevation of the test bed. The following Equation 4-3 was used to convert cathetometer readings directly into depths of flow for the skin friction tests.

$$d_f = \frac{C.R. - 12.766 \text{ cm}}{30.48 \text{ cm/ft}} \quad 4-3$$

where d_f = flume depth at selected piezometer location in ft
 C.R. = cathetometer reading in centimeters
 12.766 cm = calibration coefficient for cathetometer

Equation 4-3 was also used to determine depths for the vertical obstruction tests. Two similar equations were used for the bed form tests, but the calibration coefficients are different. These equations will be given in the following section.

Based on the recorded values for depths, discharge, and geometry of the test area, the following Equation 4-4 was used to determine the friction factor for skin friction losses. This equation is based on the Darcy-Weisbach equation and the energy equation. Both of these equations were given previously in Chapter III.

$$f'_s = \frac{2g b^2 d_m^3}{LQ^2 (1 + 2 \frac{d_m}{b})} (d_1 - d_2 + \frac{\alpha Q^2}{2g b^2} (\frac{1}{d_1^2} - \frac{1}{d_2^2})) \quad 4-4$$

where f'_s = friction factor for skin friction losses
 g = acceleration due to gravity (32.174 ft/sec², assumed)
 b = flume width (4.00 ft.)

$$d_m = \frac{d_1 + d_2}{2} = \text{average depth of flow}$$

L = length of flume between sections 1 and 2

Q = discharge

d_1 = depth of flow in upstream section 1

d_2 = depth of flow in downstream section 2

α = energy coefficient (assumed equal to 1.06)

Friction factors calculated by the use of Equation 4-4 are tabulated in the following chapter. These values are used to prepare an experimental equation giving the friction factor as a function of the average depth of flow. Note that hydraulic radius in Equation 4-4 is defined in a traditional manner, i.e., cross-sectional area of the flow divided by the wetted perimeter. Some of the other equations used in this chapter will equate hydraulic radius with average depth of flow. This will be noted.

Bed Form Setup

The geometric shape selected for the bed form tests was presented in Chapter II. In order to cover an expanded range of values for the ratio of bed-form-height to depth-of-flow two sets of bed forms were tested. Both sets had the same triangular shape; however, the scale of the forms was different. In-place dimensions of a single bed form from each set are given in Figure 4-8. For both sets of bed forms each element was numbered beginning at the upstream end of the channel. Forty-two elements were used for the large bed forms, and one-hundred-sixty-eight elements were used for the small forms. For both sets of bed forms the first piezometer was placed at about 13 ft from the upstream end of the forms. This placement allowed the flow pattern to be established in advance of the first piezometer.

For the larger bed forms the triangular roughness elements were attached by screws to the concrete bed of the flume. Each element was 4 ft in width and was constructed of 24 gauge, smooth, galvanized sheet-metal. A large sheet-metal break was used to bend the sheet-metal to the correct angles. Wooden supports were placed under the edges of each bed form to provide support and maintain the correct geometry. Support was also provided at the center of the form in some cases. Small nails were used to attach the sheet-metal to the wooden supports. Piezometers were placed on the lee side of bed forms number 6, 15, 24, 33, and 42. This placement provided a spacing of 19.630 ft between adjacent piezometers.

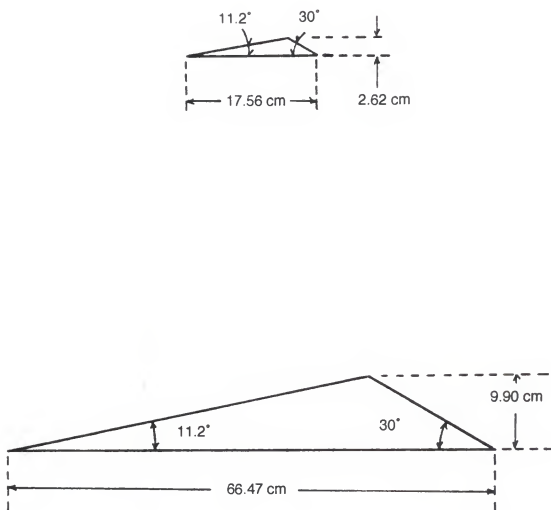


Figure 4-8: Bed Form Dimensions

The smaller bed forms were constructed and attached to the concrete in approximately the same manner as the larger forms, but wooden supports were not required. Actually the same pieces of material used for the larger forms were used to construct the smaller forms. The larger forms were simply given several additional bends. Piezometers were placed on the lee side of elements number 22, 57, 92, 127, and 162. This placement provided a spacing of 20.159 ft between adjacent piezometers.

Depths of flow for both sets of bed form tests were determined with the cathetometer. The calibration coefficients however were different. Equations 4-5 and 4-6 were used to determine depths of flow at the selected piezometer locations for the larger and smaller bed forms, respectively. It should be noted explicitly that these depths are determined relative to the average elevation of the bed. This average elevation corresponds to one-half the bed form height above the lowest point of the bed form or to one-half the bed form height below the crest of the bed form.

$$d_L = \frac{C.R. - 15.647 \text{ cm}}{30.48 \text{ cm/ft}} \quad 4-5$$

where d_L = average flume depth in feet at the selected piezometer location for the larger bed forms

C.R. = cathetometer reading in centimeters

15.647 cm = calibration coefficient for cathetometer

$$d_s = \frac{C.R. - 11.840 \text{ cm}}{30.48 \text{ cm/ft}} \quad 4-6$$

where d_s = average flume depth in feet at the selected piezometer location for the smaller bed forms

C.R. = cathetometer reading in centimeters

11.840 cm = calibration coefficient for cathetometer

For all of the bed form tests the velocity profiles both vertically and horizontally were important factors. The physical calculations as well as the theoretical developments

depend on these profiles. These profiles were determined by taking literally hundreds of velocity measurements. Velocities were observed over the crests of the bed forms and along the longitudinal midpoint of the upstream face. Both shallow and deep conditions of flow were considered for both low and high velocities. The recorded values will not be given here; however, the results of these observations will be presented in equation form.

Velocities were determined through the use of a digital counter and small Ott meter. The Ott meter propeller had a diameter of approximately 3 cm. The vertical and horizontal position of the Ott meter could be varied continuously to any position in the flow greater than about 1.5 cm from the boundary. The position of the meter relative to the bed and sides of the flume was determined with an aluminum meter-stick. Recorded positions were considered to be accurate to the nearest 0.25 cm. Velocities at any particular point in the flow were determined by averaging the velocity over a three minute period. A longer time period was used in some cases, and the consistency of velocities was determined by obtaining the velocity averages at one minute intervals.

Velocity profiles were used to determine an energy coefficient. Although it is not generally used in a control volume form, the equation which expresses conservation of energy is an integral over a control volume of all energy transfers (Hughes and Brighton, 1967). As commonly used this energy conservation relationship is reduced to one dimension with a single inflow and a single outflow. Integration of the velocity term in the control volume expression is replaced by an energy coefficient and a mean velocity expression. This integration is reflected in Equation 4-7 which defines the energy coefficient. Numerical values for this coefficient were determined by numerical integration for all of the experiments.

$$\alpha = \frac{1}{A} \int_A \left(\frac{v}{v_m} \right)^3 dA \quad 4-7$$

where α = energy coefficient
 A = cross-sectional area of the flow

v	=	velocity at a particular position in the flow
v_m	=	mean velocity over the cross-section
dA	=	differential of area

Mean velocity may also be expressed in the form of an integral. This is given as Equation 4-8. Mean velocities were determined by using the observed values of depth and discharge; however, these velocities were checked several times by numerical integration of Equation 4-8.

$$v_m = \frac{1}{A} \int_A v \, dA \quad 4-8$$

Equations 4-7 and 4-8 are expressed in a one-dimensional form. Velocity would normally be considered as a vector quantity and the expressions would be modified accordingly.

For the larger bed forms the energy coefficient at the crest of the bed form was dependent on depth of flow and is given as Equation 4-9. It should be noted that the magnitude of discharge had little effect on this relationship.

$$\alpha_{\text{crest}} = 1.04 + \frac{0.8}{d} \quad 4-9$$

where α_{crest} = energy coefficient for flow over the crests of the bed forms

d = depth of flow in centimeters relative to the average bed elevation

A similar expression was obtained for the smaller bed forms. This expression is given as Equation 4-10.

$$\alpha_{\text{crest}} = 1.035 + \frac{1.0}{d^{0.9}} \quad 4-10$$

Equations 4-9 and 4-10 are only valid for the range of depths considered during the tests. In the final analysis both sets of bed forms were considered jointly. This required a

single expression for the energy coefficient. This expression is given as Equation 4-11 and represents an average of the two previous expressions.

$$\alpha_{\text{crest}} = 1.038 + \frac{0.9}{d^{0.95}} \quad 4-11$$

Expressions were also determined for the energy coefficients at the longitudinal midpoint of the upstream face, but these expressions will not be given here.

In flow areas which are very wide relative to the depth it may be shown that the hydraulic radius required by the Darcy-Weisbach equation is equivalent to the depth of flow. In narrow flow areas the hydraulic radius defined as cross-sectional area divided by wetted perimeter is generally used as defined. This use is occasionally cumbersome and it is advantageous to use depth of flow for the hydraulic radius in narrow flow areas as well. Based on the transverse velocity profiles a modified width of flow may be defined which allows depth of flow to be used as the hydraulic radius rather than the usual ratio term. This modified width accounts for the reduction of velocity to zero at the side walls of the test flume. Moving away from one side wall of the flume toward the other side wall, the velocities rapidly approach a reasonably constant value and maintain that value until very close to the opposite wall. Measuring the transverse velocity profile at one-fourth the total depth below the water surface or at three-fourths the total depth from the bed, Equation 4-12 was used to establish the modified width of flow. This equation incidentally satisfies conditions of continuity.

$$\frac{v_{m25}}{v_{\max 25}} = \frac{b'}{b} \quad 4-12$$

where v_{m25} = transverse average velocity measured at 25% of the depth below the water surface

$v_{\max 25}$ = maximum velocity occurring for the 25% depth transverse profile

b' = modified width of flume

$$b = \text{actual width of flume (4.00 ft)}$$

Based on the transverse velocity profiles the modified flow width was determined to be 3.78 ft. This value was practically independent of the depth and discharge considered and is expressed as Equation 4-13.

$$b' = 3.78 \text{ ft} \quad 4-13$$

Since the bed of the flume was horizontal, it was not possible to establish conditions of uniform flow. Consideration was therefore given to the conditions of gradually varied flow which existed. The differential equation for gradually varied flow on a horizontal bed is given as Equation 4-14 (Lecture Notes, CWR 4202).

$$dL = \frac{1 - \frac{\alpha Q^2 B}{g A^3}}{-S_{EGL}} dd \quad 4-14$$

where	dL	=	differential of abscissa in direction of flow
	α	=	energy coefficient
	Q	=	discharge
	B	=	width of water surface
	g	=	acceleration due to gravity
	A	=	cross-sectional area of the flow
	S_{EGL}	=	slope of the energy grade line, a function of d
	dd	=	differential of depth

If it is assumed that flow over the bed forms is in the range of flow for which the Manning equation is applicable, an expression may be obtained for the slope of the energy grade line in a rectangular section. This expression is given as equation 4-15.

$$S_{EGL} = \frac{Q^2}{B^2 M^2 d^{10/3}} \quad 4-15$$

where

$$M = 8.25 \frac{\sqrt{g}}{(k)^{1/6}} = \text{a resistance coefficient applicable to the Manning equation}$$

$$d = \text{local average depth of flow}$$

Depth of flow has been used for the hydraulic radius in Equation 4-15. This use will be accounted for by using the modified flow width from Equation 4-13 for the determination of numerical values based on Equation 4-15.

Returning to Equation 4-14, the ratio term in the numerator of the expression may be given in a different form. This form requires the use of the definition of critical depth in a rectangular open channel. This definition is given as Equation 4-16.

$$d_{cr} = \sqrt[3]{\frac{\alpha_{cr} Q^2}{g B^2}} \quad 4-16$$

$$\begin{aligned} \text{where } d_{cr} &= \text{critical depth} \\ \alpha_{cr} &= \text{energy coefficient at critical depth flow conditions} \end{aligned}$$

Substituting Equations 4-15 and 4-16 into Equation 4-14, Equation 4-14 may be solved analytically. Integration of the equation from an upstream position x_1 with depth d_1 to a downstream position x_2 with depth d_2 , the result may be used to obtain an expression for the resistance coefficient. This coefficient may subsequently be used to determine the friction factor for flow over the bed forms. The expression for the resistance coefficient is given as Equation 4-17. It should be noted that flume width does not appear directly in this equation. Critical depth does appear in the expression, and the modified flow width is used to evaluate numerical values of critical depth. It is assumed that all energy coefficients which appear in Equation 4-17 are equal to the energy coefficient at the crest of the bed forms for an average depth of flow in the test flume between longitudinal positions x_1 and x_2 . This average depth of flow will be considered in the following paragraphs.

$$M = \left[\frac{g(x_2 - x_1) / (\alpha_{cr} d_{cr}^{4/3})}{\frac{3}{13} \left[\left(\frac{d_1}{d_{cr}} \right)^{13/3} - \left(\frac{d_2}{d_{cr}} \right)^{13/3} \right] - \frac{3\alpha}{4\alpha_{cr}} \left[\left(\frac{d_1}{d_{cr}} \right)^{4/3} - \left(\frac{d_2}{d_{cr}} \right)^{4/3} \right]} \right]^{1/2} \quad 4-17$$

where	M	=	Manning's resistance coefficient
	x_2	=	downstream position coordinate
	x_1	=	upstream position coordinate
	α_{cr}	=	energy coefficient at critical flow conditions
	d_{cr}	=	critical depth
	d_1	=	depth of flow at upstream position
	d_2	=	depth of flow at downstream position
	α	=	energy coefficient for the average depth of flow between positions x_1 and x_2

Equations 4-12, 4-13, 4-15, and 4-17 were all based on replacement of the hydraulic radius by channel depth. The resistance coefficient is determined as an average resistance coefficient over the specified length of flow between locations x_1 and x_2 of the flume. It is also necessary to develop an expression for the average depth of flow between these two flume locations. A mean depth for the flume between locations x_1 and x_2 may be expressed by the following Equation 4-18.

$$d_m = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} d \, dx \quad 4-18$$

where	d_m	=	mean depth of flow between positions x_1 and x_2 of the flume (longitudinal mean)
	d	=	depth of flow at any location between positions x_1 and x_2
	dx	=	differential of length

The differential of length which appears in Equation 4-18 may be obtained from Equation 4-14. Making substitutions similar to those used in the derivation of Equation 4-

17, Equation 4-18 may be solved for the mean depth of flow. This expression is given as Equation 4-19.

$$d_m = \frac{M^2}{g(x_2 - x_1)} \left[\frac{3\alpha}{7} (d_2^{7/3} - d_1^{7/3}) - \frac{3\alpha\sigma}{16 d_\alpha^3} (d_2^{16/3} - d_1^{16/3}) \right] \quad 4-19$$

It will be noted that there is a minor degree of iteration involved in solving for the resistance coefficient and mean depth. This iteration involves the exact value for the energy coefficient. Since the energy coefficient has such a small effect on the results, it is sufficient to average the upstream and downstream depths and determine an energy coefficient for this depth value.

Conversion of the resistance coefficient to a friction factor is based on the definition of resistance coefficient. This definition is given as Equation 4-20 and is expressed in terms of the variables determined in the previous equations.

$$f'_b = \frac{2g}{M^2 d_m^{1/3}} \quad 4-20$$

where f'_b = friction factor for bed forms

Equations 4-5 through 4-20 are required to convert laboratory measurements into a friction factor characteristic of the conditions considered. These equations are independent of the theoretical equations given in Chapter III. It should be noted explicitly that the friction factor determined by Equation 4-20 represents not only the energy losses due to the bed forms but the energy losses due to skin friction in the flow over these forms. Separation of these two losses is considered in Chapter V.

Vertical Obstruction Setup

All vertical obstructions were tested on the sheet-metal bed used for the skin friction tests. The piezometer spacing was 19.542 ft, and the space between adjacent piezometers was called a section. Although four sections were possible, vertical

obstructions were placed in a maximum of two sections. A single section was used in some cases. Table 4-1 indicates the number of sections used for each of the nine different arrangements of obstructions. These arrangements are indicated by test group number, one through nine. Geometric properties of the obstructions and other factors are also provided in Table 4-1. This information is considered in the following paragraphs.

All vertical obstructions were short sections of circular PVC pipe approximately 24 inches in length. The end of the pipe placed against the sheet-metal bed was cut off square to provide a good fit. Three different pipe sizes were used with nominal inside diameters of 2 inches, 3 inches, and 4 inches. The outside diameters had average values of 0.1958 ft, 0.2908 ft, and 0.3733 ft, respectively. The top ends of the pipes were held in place by small boards which spanned the width of the test flume. These small boards were secured to larger boards run down each side of the flume and anchored to the walls. The system of PVC obstructions and boards were fastened together using wire, nails, sheet-metal screws, metal brackets, and bolts. Photographs of two of the obstruction layouts and retaining system are provided as Figures 4-9 and 4-10.

The number, size, and spacing of pipes used for the nine different arrangement are indicated in Table 4-1. These arrangements are also indicated by Figures 4-11 through 4-19. In all cases the distribution being tested was duplicated in a continuous manner for a minimum distance of approximately 6 ft upstream of the first piezometer. This was done to develop the flow pattern in advance of the test area.

The distance between adjacent piezometers was 19.542 ft, and the longitudinal spacing of the rows of obstructions was adjusted such that the first and last row of pipes in each 19.542 ft section coincided with a piezometer. The centers of the pipes in the first and last rows were directly over the piezometer tubing. This implies that the piezometers were located in a section with contracted flow. This contraction is accounted for by the convergence factor given in Table 4-1. The derivation of this factor is considered in the following paragraphs.

Table 4-1. Obstruction Group Data

Test Group	Number of Pipes per Section	Number of Sections	Pipe Diameter (ft)	Section A_o (ft ²)	P_o	Lateral Spacing (ft)	Longitudinal Spacing (ft)	Convergence Factor K	Number of Tests	Layout Figure
1	6	2	.1958	.1807	.00321	1.333	9.770	.8531	3	4-11
2	12	2	.1958	.3614	.00462	1.333	4.886	.8531	3	4-12
3	24	2	.1958	.7229	.00925	1.333	2.443	.8531	3	4-13
4	48	2	.1958	1.4457	.08160	1.333	1.222	.8531	7	4-14
5	96	2	.1958	2.8915	.03699	1.333	1.222	.8531	21	4-15
6	48	2	.1958	1.4457	.08150	1.333	2.443	.8531	6	4-16
7	12	2	.3733	1.3156	.01680	1.333	4.886	.7200	10	4-17
8	39	1	.3733	4.2691	.05461	1.333	1.503	.7200	18	4-18
9	64	1	.1908	4.2515	.05439	1.0	1.222	.7092	8	4-19

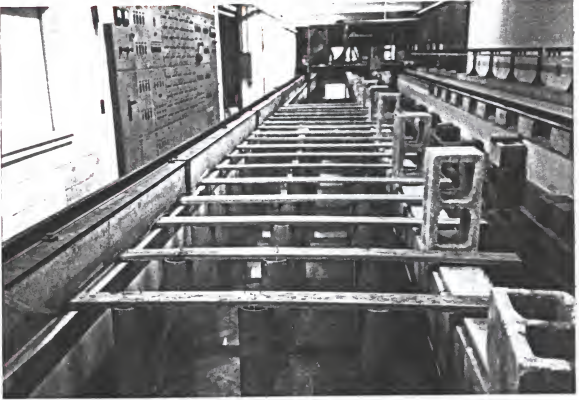


Figure 4-9: Obstruction Layout, Test Group 8



Figure 4-10: Obstruction Layout, Test Group 9

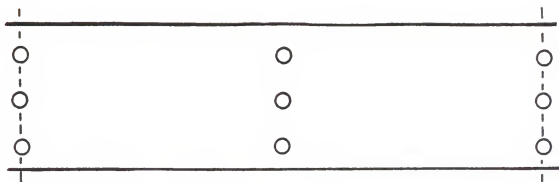


Figure 4-11: Obstruction Layout, Test Group 1

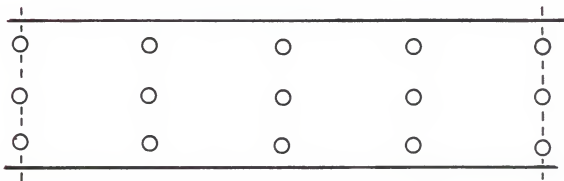


Figure 4-12: Obstruction Layout, Test Group 2

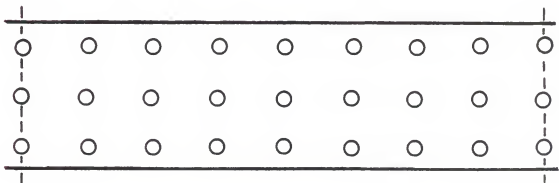


Figure 4-13: Obstruction Layout, Test Group 3

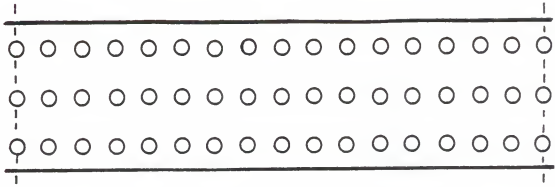


Figure 4-14: Obstruction Layout, Test Group 4

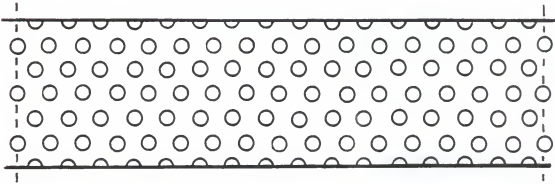


Figure 4-15: Obstruction Layout, Test Group 5

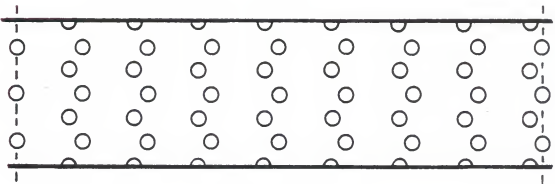


Figure 4-16: Obstruction Layout, Test Group 6

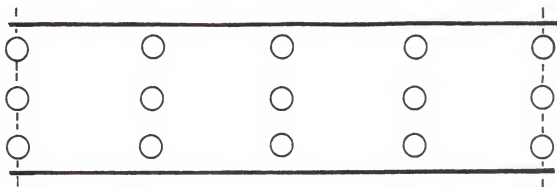


Figure 4-17: Obstruction Layout, Test Group 7

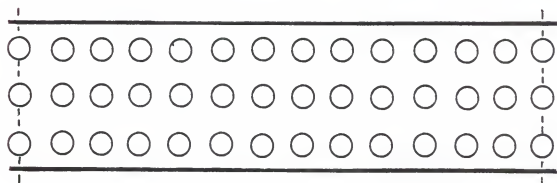


Figure 4-18: Obstruction Layout, Test Group 8

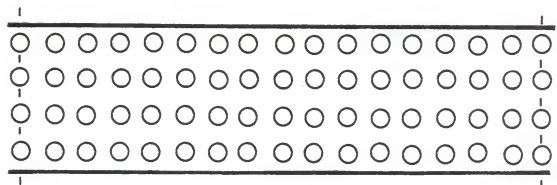


Figure 4-19: Obstruction Layout, Test Group 9

A_0 in Table 4-1 is the total cross-section area of obstructions in each section. It may be obtained by multiplying the number of pipes per section and the cross-section area of each pipe. This area is $\pi/4$ times the square of the diameter. P_0 in Table 4-1 is a dimensionless pipe density and is equal to the cross-sectional area of the pipes in the section divided by the total area of the section. All sections had an area of 4.00 ft. x 19.542 ft. = 78.168 ft². The convergence factor given in Table 4-1 is used to relate the depth of flow in the contracted section, d_c , and the depth of flow just upstream of the row of pipes, d_f . This upstream depth, d_f , represents the flume depth of flow at the specified piezometer location. The depths d_c and d_f are related by assuming constant specific energy between the section just upstream of the pipes and the contracted section. This is a reasonable assumption since the bed is horizontal, skin friction losses are very small, and the flow is converging between the two sections. The factors d_f and K are given by the following Equations 4-21 and 4-22, respectively. Note that Equation 4-21 is iterative.

$$d_f = d_c + \frac{\alpha Q^2}{2gb^2} \left(\frac{1}{K^2 d_c^5} - \frac{1}{d_f^5} \right) \quad 4-21$$

$$K = 1 - \frac{n\delta}{b} \quad 4-22$$

where	d_f	=	flume depth just upstream of contracted section
	d_c	=	depth of flow in contracted section as determined by cathetometer reading
	α	=	energy coefficient (assumed equal to 1.06)
	K	=	convergence factor
	n	=	number of pipes per row
	δ	=	pipe diameter

For each of the nine different arrangements of vertical obstructions a minimum of three and a maximum of twenty-one tests were conducted. Each test had a different flow depth and/or discharge, and the number of tests made with each group is indicated in Table 4-1.

When the bed forms were considered, it was necessary to develop equations to convert laboratory observations into other parameters. The same types of equations are necessary for the vertical obstructions. Equation 4-14 was used previously and is used again here; however, the full width of the test flume is considered rather than a modified flow width. Based on integration of Equation 4-14, the following Equations 4-23 and 4-24 may be used to determine the longitudinal mean depth of flow and friction factor for the vertical obstructions, respectively.

$$d_m = \frac{1}{f'_c} \left[\frac{\alpha}{L} (d_2^2 - d_1^2) - \frac{2gb^2}{5LQ^2} (d_2^5 - d_1^5) \right] \quad 4-23$$

$$f'_c = \frac{2\alpha}{L} (d_2 - d_1) + \frac{gb^2 (d_1^4 - d_2^4)}{2Q^2L} \quad 4-24$$

where	d_m	=	longitudinal mean depth of flow for the vertical obstructions
	f'_c	=	friction factor for the vertical obstruction tests
	L	=	length of flow region considered
	d_2	=	depth at downstream end of flow region
	d_1	=	depth at upstream end of flow region

It should be noted that the friction factor calculated by Equation 4-24 includes the effects of skin friction as well as the vertical obstructions. The subscript implies combined losses. Equations 4-23 and 4-24 both use the Darcy-Weisbach equation to express the slope of the energy grade line, and depth of flow is used for the hydraulic radius.

CHAPTER V

RESULTS OF LABORATORY EXPERIMENTS

Laboratory observations for the skin friction, bed form, and vertical obstruction tests are presented in this chapter. These observations are used with the equations given in Chapter IV to determine the parameters of the flow. Based on these parameters, equations are developed for the different friction factors. Some of these equations are easily developed, but others require a substantial amount of analysis. Information from other sources is also introduced here. This information is used not only for comparison but to provide guidelines for the current development.

Skin Friction Tests

Skin friction losses have been considered in the current experiments only to separate these losses from the losses due to bed forms and vertical obstructions. Friction factors for skin friction losses are important in the analysis of coastal flooding; however, the determination of these friction factors has not been considered here. Their existence is recognized and provisions have been made in the developed equations to include skin friction losses.

The setup of equipment for the skin friction tests and the equations necessary to evaluate the laboratory observations were given in Chapter IV. Table 5-1 gives both the observed and calculated values for these tests. Observations include the flow length, two piezometer readings, and the point gauge reading. Five piezometer readings were used for these tests to define the water surface profile; however, only two piezometer readings are given in Table 5-1. These two readings were considered to be the most accurate. In some cases an average of two adjacent piezometer readings was used, and

Table 5-1. Data Values And Computed Values For Obstruction-Free Flow

Piezo- meter #1 (cm)	Piezo- meter #2 (cm)	Point Gauge (cm)	d_1 (ft)	d_2 (ft)	Q (cfs)	L (ft)	d_m (ft)	f'_s
53.421	53.415	37.80	1.3338	1.3336	3.255	29.313	1.3337	.00092
29.594	29.409	38.11	.5521	.5460	3.335	29.313	.5491	.00215
37.075	36.973	37.82	.7975	.7942	3.260	39.064	.7959	.00286
41.044	40.992	47.25	.9278	.9261	6.219	19.542	.9269	.00114
48.100	48.080	47.10	1.1593	1.1586	6.162	19.542	1.1589	.00085
35.649	35.439	47.72	.7508	.7439	6.397	19.542	.7473	.00215
43.953	43.922	30.57	1.0232	1.0222	1.717	78.160	1.0227	.00362
23.858	23.032	30.50	.3639	.3388	1.705	78.1690	.3513	.00320
29.912	29.725	30.65	.5625	.5564	1.731	78.160	.5595	.00356

the length of the flow region is adjusted accordingly. It will be noted that some of the flow lengths given in the table are not a multiple of the measured piezometer spacing.

Nine tests were conducted with no obstructions in the flume. The friction factor calculated for each of these tests is given in the last column of Table 5-1. As expected, these friction factors vary with depth of flow. Equation 5-1 is based on the values given in Table 5-1 and represents a best-fit power expression relating depth of flow and friction factor.

$$f'_s = \left(\frac{0.0296}{d_m} \right)^{1.909} \quad 5-1$$

where f'_s = friction factor for skin friction on the sheet-metal bed

d_m = mean depth of flow

Equation 5-1 was used with the vertical obstruction tests to obtain a friction factor for the skin friction losses. This equation was also used in the "final" analysis of the bed form tests. In the "initial" analysis of the bed forms the friction factor for skin friction was based on the Reynolds number and the smooth-boundary relationship (Streeter, 1971). Friction factors determined by this relationship was subsequently dropped in favor of Equation 5-1. Equation 5-1 provided more accurate results.

Bed Form Tests

Thirty-seven tests were conducted with the larger bed forms, and each test had a different flow depth and/or discharge. Eleven tests were conducted with the smaller forms, and the depths and discharges were also varied. Tables 5-2 and 5-3 give the laboratory observations for the larger and smaller bed forms, respectively. These observations include the water temperature, the point gauge reading, and the piezometer readings. Some of the piezometer readings were considered to be in error. These readings are denoted by an asterisk in the tables and were not used for calculations.

Table 5-2. Laboratory Observations For Larger Bed Forms

Run	T (°C)	P.G (cm)	h ₁ (cm)	h ₂ (cm)	h ₃ (cm)	h ₄ (cm)	h ₅ (cm)
1	27.5	24.54	29.404	28.792	28.088	26.775	24.340
2	26.5	24.82	30.093	29.580	29.056	28.289	27.483
3	27.0	26.25	32.002	31.594	31.199	30.704	30.288
4	27.5	26.35	37.271	37.124	36.996	36.836	36.680
5	27.0	26.43	34.627	34.416	34.223	34.013	33.830
6	27.5	27.24	31.349	30.698	29.963	28.979	27.766
7	27.5	29.65	38.570	38.391	38.164	37.913	37.686
8	28.0	29.91	40.620	40.487	40.323	40.186	40.054
9	27.5	31.80	34.301	33.391	32.391	30.561	26.939
10	28.0	31.88	66.517				66.468
11	25.5	32.32	34.658	33.794	32.806	31.227	28.825
12	28.0	32.73	42.525	42.434	42.295	42.126	41.922
13	29.5	33.11	49.846	49.797	49.732	49.662	49.584
14	28.0	33.13	38.167	37.633	37.033	36.239	35.387
15	25.5	36.74	43.091	42.658	42.253	41.788	41.348
16	28.0	36.85	67.257	67.217	67.171	67.131	67.113
17	25.5	36.96	38.317	37.274	36.136	34.402	31.893
18	27.5	37.20	72.487	72.465	72.447	72.429	72.407
19	28.0	37.32	45.263	44.955	44.638	44.315	43.931
20	27.5	37.76	58.454	58.387	58.295	58.212	58.128

Table 5-2. Laboratory Observations For Larger Bed Forms (continued)

Run	T (°C)	P.G (cm)	h ₁ (cm)	h ₂ (cm)	h ₃ (cm)	h ₄ (cm)	h ₅ (cm)
21	28.0	38.60	48.431	48.261*	47.930	47.687	47.393
22	28.0	39.19	41.504	40.589	39.628	38.355	36.851
23	28.0	39.32	40.333	39.238	38.008	36.062*	35.205
24	28.0	41.55	43.797	42.986	42.014	40.907	39.613
25	29.5	42.86	51.435*	51.191	50.754	50.308	49.898
26	28.0	42.90	66.834	66.763	66.657	66.555	66.480
27	28.0	44.14	51.941	51.412	50.816	50.358	49.780
28	28.0	44.65	45.964	44.942	43.803	42.428	40.682
29	28.0	44.77	45.064	43.837	42.412	40.254	37.217
30	27.0	47.55	64.312	64.106	63.901	63.669	63.445
31	27.0	48.04	57.765	57.368	56.967	56.502	56.049
32	27.0	48.19	55.862	55.382	54.882	54.301	53.738
33	28.0	48.63	51.420	50.554	49.607	48.547	47.404
34	28.0	48.83	47.642	46.157	44.360	41.402	34.818
35	28.0	48.87	47.858	46.423	44.719	42.312	38.590
36	28.0	48.87	48.059	46.726	44.996	42.416	38.188
37	28.0	51.00	66.875	66.627	66.307	66.015	65.771

*Reading not used, considered in error

Table 5-3. Laboratory Observations For Smaller Bed Forms

Run	T (°C)	P.G (cm)	h ₁ (cm)	h ₂ (cm)	h ₃ (cm)	h ₄ (cm)	h ₅ (cm)
1	25	32.85	35.563	35.428	35.299	35.180	35.046
2	23.5	44.00	66.962	66.945	66.899	66.866	66.802
3	23.5	44.72	53.693	53.602	53.481	53.389	53.260
4	23.5	44.47	56.692	56.631	56.534	56.472	56.376
5	23.5	44.93	46.015	45.770	45.462	45.253	45.012
6	23.5	45.25	38.615	38.024	37.197	36.481	35.676
7	23.5	44.20	60.825	60.778	60.704	60.655	60.582
8	23	45.15	42.668	42.330	41.900	41.523	41.159
9	23	48.74	72.020	71.978	71.910	71.866	71.811
10	23	49.79	68.988	68.915	68.861	68.761	68.692
11	23	51.55	41.404	40.103	38.582	36.788	34.455

Temperatures were used to determine the fluid's viscosity which was in turn used to determine the Reynolds number. Based on the smooth boundary relationship, the Reynolds number was used to determine a friction factor for skin friction. It was assumed that approximately half of the bed form area was exposed to the streamlined portion of the flow. The other half of the area was within wake regions on the lee side of the bed forms. Thus approximately half of the flow was directly influenced by the effects of skin friction. Friction factors calculated by the smooth boundary relationship were thus reduced by one-half, and these values were subtracted from the combined friction factor determined by the experiments. Tables 5-4 and 5-5 indicate a friction factor for skin friction which is one-half the value determined by the smooth boundary relationship. The combined friction factor determined by the experiments is also given in the tables as well as the difference noted above.

It was also initially assumed that the energy coefficients applicable to each set of bed forms would provide better accuracy. This was not the case, and energy coefficients determined by Equation 4-11 provided the best results. Equation 4-11 represents an average of the individual expressions given by Equations 4-9 and 4-10. Tables 5-4 and 5-5 give the calculated values for the bed form experiments and use the individual energy coefficients and smooth-boundary friction factors.

Some additional comments concerning tables 5-4 and 5-5 are necessary. The depth $\bar{d}$ represents an average of the depths at the most upstream and most downstream piezometer locations used. This average depth was used to determine the energy coefficient at the crests of the bed forms. It was assumed that the energy coefficient at critical depth was equal to the energy coefficient at the bed form crests. The modified flume width was used to determine the critical depth and the Reynolds number. Equations for critical depth, Manning's resistance coefficient, longitudinal-mean depth, and combined friction factor were given in Chapter IV. Note that the combined friction factor is denoted $f'_b + f'_s$ in the tables. The friction factor for the bed forms exclusive of skin friction is

Table 5-4. Calculated Flow Data For Larger Bed Forms

Run	Q (cfs)	$\bar{d}$ (ft)	α_{cr}	d_{cr} (ft)	M (ft ^{1/3} /s)	d_m (ft)	$f_b' + f_s'$	$\frac{Re}{v_m d_m}$	$f_s'/2$	f_b'	$f_{b-L}/2a$	2a/d
1	.856	.368	1.15	.122	25.66	.389	.1339	16.2	.0098	.1241	.833	.835
2	.889	.431	1.13	.125	29.10	.436	.1002	16.8	.0097	.0905	.608	.745
3	1.065	.508	1.12	.140	32.77	.510	.0750	20.1	.0094	.0656	.441	.637
4	1.078	.700	1.10	.141	33.02	.700	.0665	20.4	.0094	.0571	.384	.464
5	1.088	.610	1.11	.142	36.20	.610	.0579	20.6	.0094	.0485	.326	.532
6	1.198	.456	1.13	.152	30.51	.465	.0893	22.6	.0092	.0801	.538	.699
7	1.562	.738	1.10	.180	35.97	.738	.0551	29.5	.0088	.0463	.311	.440
8	1.605	.810	1.09	.183	39.45	.810	.0444	30.3	.0088	.0356	.239	.401
9	1.938	.551	1.11	.209	30.96	.559	.0815	36.6	.0083	.0732	.491	.581
10	1.953	1.668	1.06	.207	48.70	1.668	.0229	36.9	.0083	.0146	.098	.195
11	2.036	.528	1.12	.216	32.04	.548	.0766	38.5	.0082	.0684	.459	.593
12	2.115	.872	1.09	.220	44.66	.872	.0338	40.0	.0082	.0256	.172	.372
13	2.191	1.118	1.08	.224	46.20	1.118	.0291	41.4	.0081	.0211	.142	.291
14	2.195	.693	1.10	.226	31.84	.697	.0716	41.5	.0081	.0635	.426	.466
15	2.992	.872	1.09	.277	37.45	.873	.0480	56.5	.0077	.0403	.271	.372
16	3.019	1.691	1.06	.276	39.80	1.691	.0341	57.0	.0077	.0264	.177	.192
17	3.046	.638	1.10	.281	33.72	.659	.0651	57.6	.0077	.0574	.385	.493
18	3.105	1.864	1.06	.281	50.43	1.865	.0206	58.7	.0076	.0130	.087	.174
19	3.135	.950	1.08	.285	38.81	.950	.0434	59.2	.0076	.0358	.240	.342

Table 5-4. Calculated Flow Data For Larger Bed Forms (Continued)

Run	Q (cfs)	$\bar{d}$ (ft)	α_{cr}	d_{cr} (ft)	M (ft ^{1/3} /sec)	d_m (ft)	$f_b' + f_s'$	Re ($\times 10^{-3}$) $\frac{v_m d_m}{\nu}$	$f_g/2$	f_b'	$f_{b-p}/2a$	2a/d
20	3.245	1.399	1.07	.290	42.22	1.399	.0323	61.3	.0074	.0249	.167	.232
21	3.462	1.059	1.08	.304	40.48	1.059	.0385	65.4	.0073	.0312	.209	.307
22	3.620	.772	1.09	.315	34.39	.781	.0591	68.4	.0073	.0518	.348	.416
23	3.656	.726	1.10	.317	36.88	.737	.0524	69.1	.0073	.0451	.303	.441
24	4.295	.855	1.09	.352	36.42	.861	.0510	81.2	.0071	.0439	.295	.377
25	4.699	1.145	1.08	.372	37.61	1.145	.0435	88.8	.0069	.0366	.246	.284
26	4.712	1.674	1.06	.372	43.69	1.674	.0284	89.0	.0069	.0215	.144	.194
27	5.116	1.155	1.08	.394	36.11	1.157	.0470	96.7	.0068	.0402	.270	.281
28	5.288	.908	1.09	.404	36.39	.918	.0500	99.9	.0068	.0432	.290	.354
29	5.329	.836	1.09	.407	34.65	.860	.0564	100.7	.0068	.0504	.338	.378
30	6.332	1.582	1.07	.453	41.44	1.583	.0322	119.7	.0065	.0257	.173	.205
31	6.520	1.354	1.07	.463	39.67	1.354	.0370	123.2	.0065	.0305	.205	.240
32	6.578	1.285	1.07	.466	39.41	1.286	.0381	124.3	.0065	.0316	.212	.253
33	6.750	1.108	1.08	.474	38.19	1.112	.0426	127.6	.0065	.0361	.242	.292
34	6.829	.839	1.09	.480	32.93	.905	.0545	129.0	.0064	.0481	.323	.359
35	6.846	.905	1.09	.480	36.50	.937	.0494	129.3	.0064	.0430	.289	.347
36	6.845	.901	1.09	.480	35.51	.938	.0521	129.3	.0064	.0457	.307	.346
37	7.724	1.663	1.06	.517	41.40	1.663	.0317	146.0	.0063	.0254	.171	.195

Table 5-5. Calculated Flow Data For Smaller Bed Forms

Run	Q (cfs)	$\bar{d}$ (ft)	α_{cr}	d_{cr} (ft)	M (ft ^{1/2} /sec)	d_m (ft)	$t_b' + t_s'$	Re ($\times 10^{-3}$) $\frac{v_m d_m}{\nu}$	$t_s'/2$	t_b'	$t_{bL}/2a$	2a/d
1	2.139	.770	1.09	.222	61.11	.770	.0188	40.4	.0081	.0107	.072	.111
2	5.069	1.806	1.06	.390	62.40	1.806	.0136	95.8	.0068	.0068	.046	.048
3	5.312	1.366	1.07	.403	63.80	1.366	.0142	100.4	.0067	.0075	.050	.063
4	5.227	1.466	1.07	.399	65.12	1.466	.0134	98.8	.0068	.0066	.044	.059
5	5.384	1.104	1.08	.408	61.30	1.105	.0166	101.7	.0067	.0099	.066	.078
6	5.495	.830	1.09	.415	61.17	.833	.0183	103.8	.0067	.0116	.078	.103
7	5.136	1.603	1.07	.394	62.72	1.603	.0140	97.1	.0068	.0072	.048	.054
8	5.460	.986	1.08	.412	61.92	.987	.0169	103.2	.0067	.0102	.068	.087
9	6.793	1.971	1.06	.474	63.36	1.971	.0128	128.4	.0064	.0064	.043	.044
10	7.217	1.870	1.06	.494	61.88	1.870	.0136	136.4	.0064	.0072	.048	.046
11	7.962	.856	1.09	.531	58.03	.877	.0200	150.5	.0063	.0137	.092	.098

denoted f'_b . This friction factor is used with the system geometry to evaluate the two dimensionless parameters given in the last two columns of Tables 5-4 and 5-5. These two parameters are plotted in Figure 5-1 for both sets of bed forms. Data points for the larger bed forms are indicated by small circles. Data points for the smaller bed forms are indicated by small squares. Recall that the larger bed forms had a length of 66.47 cm and a height of 9.90 cm. The smaller bed forms had a length of 17.56 cm and a height of 2.62 cm. The parameter "a" represents one-half the bed form height.

The larger and smaller bed forms considered two different sets of values for the height-to-depth ratio. Larger values were considered by the larger bed forms, and smaller values were considered by the smaller bed forms. As indicated in Figure 5-1 the results of these experiments are continuous. There is, however, a significant but reasonable amount of scatter in the data points. It was assumed that this scatter was due to the values used for the influence of skin friction. The physical order of the tests placed the bed form testing first. Evaluation of these tests was well under way before the skin friction tests were conducted. The skin friction tests revealed that the initial values of friction factors for skin friction could be improved.

Comparison of the experimental results with the theoretical equations indicates that these equations are highly dependent on the energy coefficients and the numerical coefficient ϕ . To consider both sets of bed forms simultaneously, a single expression for the energy coefficient was required. This expression was given as Equation 4-11. A single expression cannot be used for the coefficient ϕ .

The bed forms used in the current experiments were considered to be steep, and the length of the zone of separation was approximately equal to four times the bed form height (Engel, 1981). Based on this length and the geometries of the bed forms, expressions were developed for the numerical coefficient ϕ . These expressions are given for the larger and smaller bed forms, respectively, by the following Equations 5-2 and 5-3.

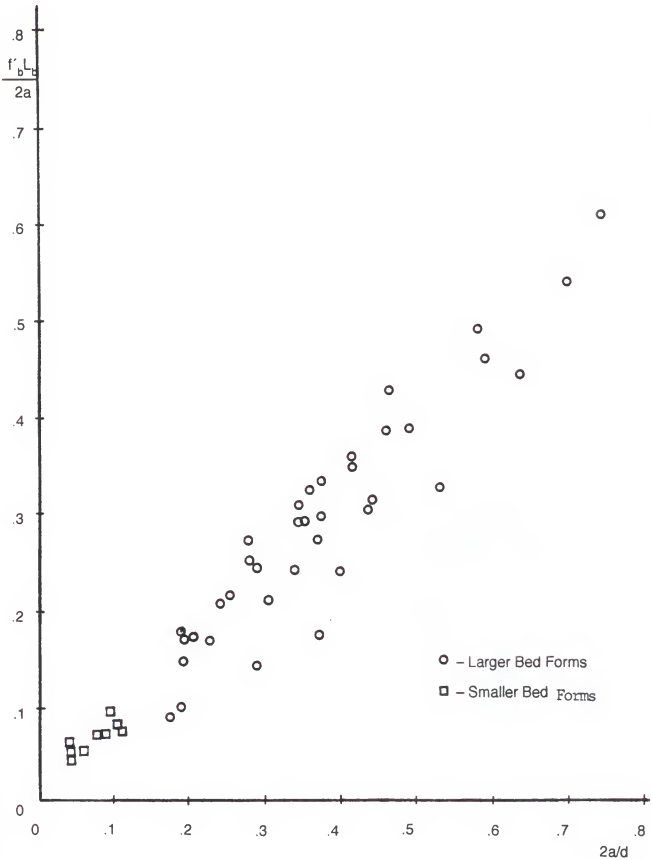


Figure 5-1: Bed Form Parameters $f'_b L_b / 2a$ vs. $2a/d$

$$\phi_L = 1 + \frac{0.50 \text{ cm}}{d} \quad 5-2$$

where ϕ_L = numerical coefficient for the larger bed forms
 d = mean depth of flow in centimeters.

$$\phi_S = 1 + \frac{0.12 \text{ cm}}{d} \quad 5-3$$

where ϕ_S = numerical coefficient for the smaller bed forms
 d = mean depth of flow in centimeters

The value of the numerical coefficient determined by either of these equations has the effect of defining the downstream cross-sectional area of the expanding region of the flow. Thus this value greatly effects the magnitude of the energy loss determined by the theoretical expression.

Using Equation 4-11 for the energy coefficients and Equation 5-1 for the friction factors for skin friction, new numerical values for the dimensionless parameters $f'_b L_b / 2a$ and $2a/d$ were calculated. These new values are tabulated in Table 5-6 and are plotted in Figure 5-2. No distinction is made between the larger and smaller bed forms. There is still some scatter in the data points, but the results appear to be slightly better than those given in Figure 5-1.

Figure 5-2 uses arithmetic scales on the axes, and there is a definite curvature in the plotted points. A best-fit power expression is used to represent the data points and is sketched in Figure 5-2. This power expression is given by the following Equation 5-4.

$$\frac{f'_b L_b}{2a} = 0.062 + 0.85 \left(\frac{2a}{d} \right)^{1.15} \quad 5-4$$

where f'_b = friction factor for bed forms exclusive of skin friction
 L_b = length of the bed form
 a = one-half the bed form height
 d = mean depth of flow over bed form

Table 5-6. New Values For Bed Form Parameters

Run	$\frac{f_{1-b}}{2a}$	2a/d	Run	$\frac{f_{1-b}}{2a}$	2a/d
1	.8509	.8355	25	.2857	.2836
2	.6337	.7451	26	.1876	.1941
3	.4745	.6366	27	.3096	.2808
4	.4303	.4641	28	.3265	.3539
5	.3679	.5325	29	.3681	.3775
6	.5647	.6986	30	.2125	.2052
7	.3552	.4402	31	.2436	.2398
8	.2858	.4009	32	.2509	.2526
9	.5233	.5815	33	.2794	.2920
10	.1505	.1947	34	.3572	.3590
11	.4894	.5929	35	.3229	.3468
12	.2163	.3725	36	.3415	.3465
13	.1885	.2906	37	.2097	.1953
14	.4649	.4662	1	.1129	.1115
15	.3120	.3721	2	.0885	.0475
16	.2260	.1921	3	.0912	.0628
17	.4193	.4931	4	.0858	.0585
18	.1356	.1743	5	.1046	.0777
19	.2828	.3418	6	.1114	.1029
20	.2125	.2322	7	.0906	.0535
21	.2515	.3067	8	.1049	.0869
22	.2515	.4160	9	.0836	.0435
23	.3375	.4405	10	.0891	.0459
24	.3318	.3771	11	.1241	.0979

The current data will be represented by Equation 5-4, and a graph of this equation will be used for comparison with the theoretical equations and for comparison with experimental data from other sources.

Wang and Shen (1980) conducted a series of laboratory tests with an alluvial bed. They considered the geometry of the bed forms and the energy losses. Their experimental results are expressed in the current form and are tabulated in Table 5-7. Values from Table 5-7 are plotted in Figure 5-3. These data points are indicated by small squares. These results compare favorably with the current experiments. A graph of Equation 5-4 is sketched in Figure 5-3.

Shen, Fehman, and Mendoza (1980) performed a series of laboratory experiments using model bed forms similar to the bed forms used for the current tests. Their results are expressed in the current form and are tabulated in Table 5-8. Values from Table 5-8 are also plotted in Figure 5-3 and are denoted by small triangles. These results also compare favorably with the current results.

The theoretical equation developed previously is given again here as Equation 5-5. Subscripts one and two imply cross-sections one and two, respectively.

$$\frac{f'_b L_b}{2a} = \frac{1}{2} \frac{1}{a/d} \left(\frac{\alpha_1}{F^2} - \frac{2\alpha'_1}{\phi F} + \frac{2\alpha'_2 - \alpha_2}{\phi^2} \right) \quad 5-5$$

where

$$F = \left(1 - \frac{\varepsilon}{d} - \frac{a}{d} \right)$$

$$\varepsilon = \frac{d}{2} \left(\frac{Fr_m}{\phi} \right)^2 \left(\frac{\alpha_1 \phi^2}{F^2} - \alpha_2 \right)$$

$$Fr_m = \frac{v_m}{\sqrt{gd_m}} = \text{Froude number}$$

f'_b = friction factor for bed forms

L_b = bed form length

a = one-half the bed form height

d	=	mean depth of flow
α	=	energy coefficient
α'	=	momentum coefficient
ϕ	=	dimensionless numerical coefficient
ε	=	water surface depression
v_m	=	mean velocity
g	=	acceleration due to gravity

Table 5-9 is developed to consider the theoretical equation under specific conditions. It will be assumed that the larger bed forms are used and that the Froude number is equal to 0.10. Depth will be used as a parameter and a reasonable range of values is considered. Unless otherwise specified, energy coefficients are calculated using Equation 4-11. The dimensionless numerical coefficient is calculated using Equation 5-2. Recall that the bed form length is 66.47 cm and the height is 9.90 cm. It is assumed that the energy coefficients at cross-sections one and two are equal.

Two sets of values for the friction factor parameter are given in Table 5-9. These values are also plotted in Figure 5-3. The parameters obtained using the laboratory values for the energy coefficient are low but are reasonably close to the experimental results. These values are represented by small circles in Figure 5-3. The values obtained using the Carnot approximation are far too low. These values are represented by small hexagons. Thus the energy coefficients are indeed important factors. It should be noted that other relationships could be used to define the dimensionless numerical coefficient ϕ . Although it will not be shown here, the limits defined earlier may be used for ϕ , and two lines would be produced which enclose the experimental values. Thus a ϕ value could be selected which would make the theoretical equation match the experimental results. Such a value could be found for the Carnot energy coefficients or the laboratory values. This ϕ value would define the point of reattachment of the streamline separating the wake region from the main flow. It is assumed that the energy coefficients and the ϕ value are defined

Table 5-7. Data Presented by Wang and Shen Plus Additional Parameters

Group	Mean Dune Height (ft)	Mean Dune Length (ft)	Friction Factor f_b	Mean Flow Depth (ft)	$f_b' L_b / 2a$	$2a/d$
1	.160	3.975	.00525	.962	.130	.166
2	.146	3.844	.00575	.955	.151	.153
3	.147	3.781	.00750	.966	.193	.152
4	.158	3.918	.00800	.962	.198	.164
5	.155	3.963	.00875	.956	.224	.162
6	.162	3.898	.00950	.953	.229	.170
7	.186	4.287	.01100	.962	.254	.193
8	.183	4.142	.01250	.952	.283	.192
9	.202	4.362	.02550	.997	.551	.203
10	.254	4.275	.01575	1.029	.265	.247
11	.265	4.363	.01725	1.025	.284	.259
12	.276	4.539	.01875	1.033	.308	.267
13	.270	4.597	.01900	1.015	.323	.266
14	.309	4.684	.01900	1.076	.288	.287
15	.309	4.475	.02125	1.060	.308	.292
16	.300	4.406	.02400	1.067	.352	.281
17	.363	5.431	.02225	1.141	.333	.318
18	.407	5.821	.02450	1.186	.350	.343
19	.389	5.740	.02500	1.174	.369	.331
20	.369	5.288	.02375	1.147	.340	.322
21	.376	5.426	.02425	1.161	.350	.324
22	.374	5.816	.02450	1.129	.381	.331

Table 5-8. Data Presented By Shen, Fehlman and Mendoza
Plus Additional Paramaters

Run	f'_b	d (cm)	$f'_b L_b / 2a$	$2a/d$
1	.10825	22.1	.723	.620
2	.07725	27.2	.516	.504
3	.05900	32.3	.394	.424
4	.10500	22.1	.701	.620
5	.08125	27.2	.543	.504
6	.06925	32.3	.463	.424
7	.10200	22.1	.681	.620
8	.08675	27.2	.579	.504
9	.06575	32.3	.439	.424
10	.13375	22.1	.893	.620
11	.08625	27.2	.576	.504
12	.07300	32.3	.488	.424
1R	.10700	22.1	.715	.620
2R	.06350	27.2	.424	.504
3R	.05750	32.3	.384	.424
7R	.09425	22.1	.629	.620
8R	.08150	27.2	.544	.504
9R	.05800	32.3	.387	.424

Table 5-9. Theoretical Values for $f'_b L_b / 2a$

$2a/d$	d (cm)	ϕ	α	α'	$\frac{\epsilon}{d}$	F	$\frac{f'_b L_b}{2a}$	$\frac{f'_b L_b}{2a}^*$
.05	198	1.0025	1.044	1.015	.0003	.9747	.0498	.0162
.10	99	1.0051	1.049	1.016	.0006	.9494	.0741	.0341
.15	66	1.0076	1.055	1.018	.0010	.9240	.1007	.0536
.20	40.5	1.0101	1.060	1.020	.0014	.8986	.1286	.0753
.30	33	1.0152	1.070	1.023	.0023	.8477	.1952	.1259
.50	19.8	1.0253	1.091	1.033	.0046	.7454	.3798	.2672
.75	13.2	1.0379	1.116	1.039	.0096	.6154	.7819	.5783

* $\delta = 0, \alpha = \alpha' = 1.0$ (Carnot approximation)

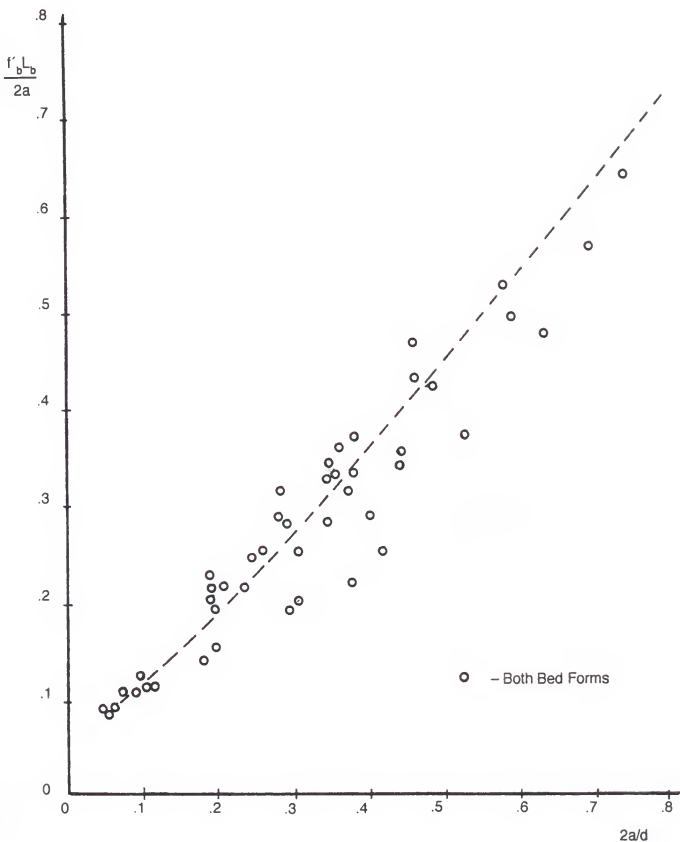


Figure 5-2: Revised Bed Form Parameters $f'_b L_b / 2a$ vs. $2a/d$

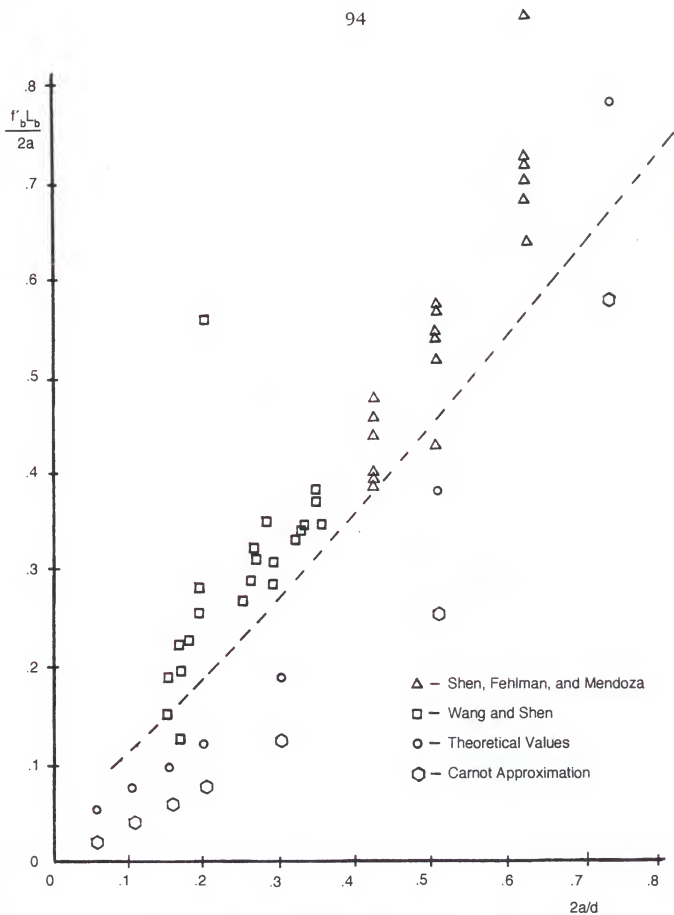


Figure 5-3: Comparison Bed Form Parameters $f'_b L_b / 2a$ vs. $2a/d$

by the flow and bed forms considered. Rather than specify exact values for α and ϕ the experimental equation will be used.

Equation 5-4 is the experimentally derived equation relating the dimensionless parameters $f'_{bL_b}/2a$ and $2a/d$. It does not contain energy coefficients or the factor ϕ . These variables are implicitly included in the equation. It is assumed that Equation 5-4 will provide reasonable estimates of the friction factor for subcritical flows. This friction factor is based on the geometry of the bed forms and the depth of flow.

Vertical Obstruction Tests

All of the vertical obstruction tests were conducted on the sheet-metal bed used for the skin friction tests. Obstructions could be secured to the sheet-metal without damage to the concrete bed of the flume.

Nine different arrangements of vertical obstructions were tested, and each arrangement was given a test group number - one through nine. Individual tests within each test group were also numbered. Table 4-1 given previously contains the general obstruction group data. This table is provided again here as Table 5-10. Laboratory observations and calculated values are given for all test groups in the following Tables 5-11 through 5-19. Note specifically that some of the numerical values in the tables are given to an unrealistic accuracy. This is done to avoid round-off errors.

Equations were given in Chapter IV to determine the flow parameters based on the laboratory observations. Additional equations will be given in this chapter, and comments will be made in the following paragraphs concerning the development of Tables 5-11 through 5-19.

Although a single value is given in the tables for the cathetometer reading for a particular piezometer, the value given represents the results of many readings. In some cases a simple mean was used, and in other cases the readings were plotted against time and an appropriate value was selected. Some of the tests required as much as four hours for the water surface elevations to stabilize. When stable or reasonably so, minor

Table 5-10. Obstruction Group Data

Test Group	Number of Pipes per Section	Number of Sections	Pipe Diameter (ft)	Section A_o (ft ²)	P_o	Lateral Spacing (ft)	Longitudinal Spacing (ft)	Convergence Factor K	Number of Tests	Layout Figure
1	6	2	.1958	.1807	.00321	1.333	9.770	.8531	3	4-11
2	12	2	.1958	.3614	.00462	1.333	4.886	.8531	3	4-12
3	24	2	.1958	.7229	.00925	1.333	2.443	.8531	3	4-13
4	48	2	.1958	1.4457	.08160	1.333	1.222	.8531	7	4-14
5	96	2	.1958	2.8915	.03699	1.333	1.222	.8531	21	4-15
6	48	2	.1958	1.4457	.08150	1.333	2.443	.8531	6	4-16
7	12	2	.3733	1.3156	.01680	1.333	4.886	.7200	10	4-17
8	39	1	.3733	4.2691	.05461	1.333	1.503	.7200	18	4-18
9	64	1	.1908	4.2515	.05439	1.0	1.222	.7092	8	4-19

Table 5-11. Test Group 1; $\delta = .1958$ ft; $P_0 = .0023$ ft²; $K = .8531$

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
1	27.474	27.233	27.022	30.88	.0127	27	39.084
	.4825		.4677	1.771	.0049	12880	.4809
	.4880		.4736	.9206	.0078	1.144	.0083
2	45.267	45.225	45.179	30.64	.0309	27	39.084
	1.0663		1.0634	1.729	.0011	5670	1.0659
	1.0673		1.0644	.4055	.0298	9.59	.0154
3	37.482	37.389	37.315	31.11	.0230	27	39.084
	.8109		.8054	1.812	.0018	7820	.8101
	.8128		.8074	.5591	.0212	1.034	.0126

Table 5-12. Test Group 2; $\delta = .1958$ ft; $P_0 = .0046$ ft²; $K = .8531$

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
1	41.175	41.068	40.964	31.21	.0433	27.5	39.084
	.9321		.9251	1.830	.0014	6880	.9301
	.9335		.9267	.4918	.0419	1.004	.0281
2	26.460	25.921	25.439	31.18	.0200	27.5	39.084
	.4493		.4158	1.824	.0058	14480	.4407
	.4561		.4240	1.0350	.0142	1.169	.0155
3	33.541	33.328	33.120	31.22	.0327	27.5	39.084
	.6816		.6678	1.832	.0025	9450	.6777
	.6845		.6707	.6757	.0302	1.078	.0220

Table 5-13. Test Group 3; $\delta = .1958$ ft; $P_0 = .0093$ ft²; $K = .8531$

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
1	34.430	34.135	33.835	31.34	.0508	27.5	39.084
	.7108		.6912	1.853	.0024	9210	.7039
	.7134		.6941	.6582	.0484	1.072	.0454
2	25.344	24.318	23.220	31.29	.0262	27.5	39.084
	.4127		.3430	1.844	.0072	16460	.3918
	.4212		.3563	1.1768	.0190	1.196	.0282
3	46.523	46.426	46.333	30.29	.0792	27.5	39.084
	1.1075		1.1013	1.669	.0010	5280	1.1053
	1.1084		1.1022	.3775	.0782	.943	.0627

Table 5-14. Test Group 4; $\delta = .1958$ ft; $P_o = .0185$ ft²; $K = .8531$

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
1	46.571	46.429	46.275	30.22	.1252	27.5	39.084
	1.1091		1.0994	1.657	.0010	5240	1.1051
	1.1099		1.1002	.3748	.1242	.941	.1251
2	29.060	28.193	27.297	31.31	.0553	27.5	39.084
	.5346		.4767	1.848	.0043	12600	.5128
	.5394		.4829	.9009	.0510	1.140	.0703
3	38.103	37.770	37.437	31.40	.0905	27.5	39.084
	.8313		.8094	1.864	.0017	7930	.8225
	.8332		.8115	.5666	.0888	1.037	.1026
4	52.907	52.782	52.661	31.51	.1352	28	39.084
	1.3170		1.3089	1.884	.0007	5020	1.3137
	1.3178		1.3097	.3586	.1345	.931	.1471
5	52.871	52.129	51.444	42.31	.1279	28	39.084
	1.3158		1.2690	4.527	.0007	12200	1.2976
	1.3204		1.2740	.8721	.1271	1.133	.1769
6	53.279	51.741	50.413	47.86	.1215	28	39.084
	1.3292		1.2351	6.450	.0007	17430	1.2941
	1.3386		1.2461	1.2461	.1208	1.208	.1880
7	53.817	53.435	53.078	37.97	.1384	28	39.084
	1.3468		1.3226	3.299	.0007	8630	1.3372
	1.3491		1.3250	.6167	.1377	1.057	.1700

Table 5-15. Test Group 5; $\delta = .1958$ ft; $P_0 = .0370$ ft²; $K = .8531$

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
1	41.485	40.263	38.771	34.89	.2546	29	39.084
	.9422		.8532	2.565	.0015	9930	.9031
	.9451		.8568	.7100	.2532	1.090	.2368
2	40.644	39.161	37.401	34.89	.2686	29	39.084
	.9146		.8082	2.565	.0016	10330	.8682
	.9177		.8122	.7386	.2671	1.098	.2294
3	44.235	43.165	42.030	34.82	.2871	29	39.084
	1.0324		.9601	2.550	.0012	8910	1.0001
	1.0348		.9629	.6373	.2859	1.065	.2561
4	47.162	44.232	40.609	43.81	.2336	29	39.084
	1.1285		.9135	5.006	.0011	16800	1.0423
	1.1364		.9259	1.2008	.2325	1.200	.3008
5	50.986	48.349	45.377	43.58	.3066	29	39.084
	1.2539		1.0699	4.931	.0009	19660	1.1765
	1.2601		1.0784	1.0478	.3057	1.171	.3314
6	60.492	59.040	57.524	43.32	.3774	28.5	39.084
	1.5658		1.4684	4.846	.0005	11130	1.5227
	1.5695		1.4727	.7957	.3768	1.114	.4079
7	56.644	54.584	52.193	43.46	.4049	28.5	39.084
	1.4396		1.2935	4.892	.0007	12440	1.3755
	1.4441		1.2992	.8891	.4043	1.137	.3762

Table 5-15-continued

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_e Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
8	62.902	60.417	57.572	48.53	.3799	28.5	39.084
	1.6449		1.4700	6.711	.0005	14950	1.5697
	1.6514		1.4783	1.0687	.3794	1.176	.4438
9	58.284	55.431	52.013	48.87	.3036	28.5	39.084
	1.4934		1.2876	6.845	.0006	17000	1.4079
	1.5017		1.2990	1.2154	.3029	1.202	.4071
10	54.766	51.269	47.027	48.80	.2734	28.5	39.084
	1.3779		1.1241	6.817	.0008	18680	1.2762
	1.3877		1.1392	1.3355	.2726	1.222	.3751
11	51.265	46.952	41.316	49.19	.2259	28.5	39.084
	1.2631		.9367	6.973	.0009	21360	1.1416
	1.2754		.9609	1.5271	.2249	1.250	.3432
12	49.580	44.704	37.820	49.20	.2088	28.5	39.084
	1.2078		.8220	6.977	.0011	22740	1.0729
	1.2214		.8554	1.6258	.2078	1.263	.3259
13	64.787	64.508	64.205	33.78	.4528	28	39.084
	1.7067		1.6876	2.327	.0004	4790	1.6980
	1.7074		1.6884	.3427	.4523	.920	.3757
14	66.901	66.158	65.386	40.24	.4574	28	39.084
	1.7761		1.7264	3.912	.0004	7800	1.7535
	1.7780		1.7284	.5577	.4570	1.034	.4359

Table 5-15-continued

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
15	65.636	64.173	62.666	44.98	.4177	28	39.084
	1.7346		1.6371	5.401	.0004	11170	1.6913
	1.7384		1.6414	.7984	.4173	1.115	.4534
16	67.842	65.239	62.561	50.61	.4001	28	39.084
	1.8070		1.6337	7.559	.0004	15260	1.7323
	1.8138		1.6422	1.0908	.3997	1.180	.4915
17	67.310	63.878	59.980	53.78	.3575	28	39.084
	1.7895		1.5490	8.970	.0004	18560	1.6896
	1.7995		1.5625	1.3272	.3270	1.221	.4961
18	32.180	31.306	30.354	28.85	.1692	28	39.084
	.6369		.5770	1.435	.0031	8220	.6107
	.6389		.5795	.5873	.1661	1.046	.1536
19	34.057	32.347	29.942	33.31	.1756	28	39.084
	.6985		.5635	2.231	.0028	12130	.6434
	.7026		.5699	.8669	.1728	1.132	.1751
20	38.720	35.848	32.040	39.30	.1711	28	39.084
	.8515		.6323	3.650	.0020	16610	.7685
	.8589		.6467	1.1874	.1691	1.198	.2213
21	43.498	39.818	34.914	43.60	.1913	28	39.084
	1.0083		.7266	4.937	.0015	19090	.9044
	1.0180		.7470	1.3648	.1898	1.227	.2668

Table 5-16. Test Group 6; $\delta = .1958$ ft; $P_o = .0185$ ft²; $K = .8531$

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
1	38.805	37.908	36.949	35.10	.1290	27.5	39.084
	.8543		.7934	2.612	.0017	11020	.8290
	.8580		.7977	.7876	.1273	1.112	.1109
2	40.035	38.032	35.613	41.06	.1178	27.5	39.084
	.8946		.7496	4.149	.0017	17290	.8392
	.9034		.7624	1.2360	.1162	1.206	.1217
3	42.167	37.919	30.733	47.72	.1025	27.5	39.084
	.9646		.5895	6.397	.0016	26090	.8575
	.9832		.6591	1.8649	.1009	1.292	.1332
4	37.791	37.301	36.785	31.22	.1336	27.5	39.084
	.8210		.7880	1.832	.0018	7940	.8069
	.8230		.7901	.5675	.1317	1.038	.1007
5	37.157	36.668	36.140	31.08	.1282	27.5	39.084
	.8002		.7669	1.806	.0019	8040	.7860
	.8022		.7690	.5746	.1262	1.041	.0984
6	36.707	36.571	36.372	25.70	.1384	27.5	39.084
	.7855		.7745	.995	.0019	4460	.7806
	.7861		.7751	.3185	.1365	.903	.0848

Table 5-17. Test Group 7; $\delta = .3733$ ft; $P_0 = .0168$ ft²; $K = .7200$

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
1	27.869	25.790	23.032	35.11	.0386	27	39.084
	.4955		.3368	2.614	.0049	36470	.4778
	.5253		.4157	1.3677	.0337	1.362	.0373
2	36.985	36.435	35.928	34.75	.0658	27	39.084
	.7946		.7599	2.534	.0019	21440	.7881
	.8046		.7709	.8039	.0639	1.251	.0565
3	65.547	65.481	65.397	32.29	.1630	27	39.084
	1.7317		1.7267	2.030	.0004	7820	1.7305
	1.7330		1.7281	.2933	.1626	1.034	.1026
4	59.257	59.125	58.936	34.25	.1655	27	39.084
	1.5253		1.5148	2.426	.0005	10620	1.5225
	1.5277		1.5172	.3984	.1650	1.104	.0963
5	45.652	45.338	45.037	34.58	.1034	27	39.084
	1.0789		1.0588	2.497	.0011	15500	1.0742
	1.0841		1.0641	.5812	.1023	1.183	.0728
6	56.867	56.359	55.982	42.17	.1110	27	39.084
	1.4469		1.4179	4.483	.0006	20730	1.4420
	1.4562		1.4276	.7773	.1104	1.244	.1028
7	53.465	52.804	52.276	43.12	.1013	27	39.084
	1.3353		1.2963	4.782	.0007	23990	1.3289
	1.3478		1.3096	.8996	.1006	1.274	.0970

Table 5-17-continued

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
8	46.353	45.376	44.628	43.05	.0798	27	39.084
	1.1019		1.0453	4.760	.0010	29010	1.0939
	1.1204		1.0660	1.0878	.0788	1.314	.0824
9	38.692	36.733	34.938	43.33	.0640	27	39.084
	.8506		.7274	4.850	.0017	38780	.8337
	.8842		.7754	1.4543	.0623	1.375	.0657
10	36.249	33.536	30.521	43.43	.0582	27	39.084
	.7704		.5825	4.882	.0021	43510	.7481
	.8131		.6668	1.6315	.0561	1.385	.0594

Table 5-18. Test Group 8; $\delta = .3733$ ft; $P_0 = .0546$ ft²; $K = .7200$

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
1	35.756	35.509		25.52	.1900	25	19.542
	.7543	.7462		.972	.0021	8620	.7518
	.7559	.7478		.3233	.1880	1.057	.1480
2	52.026	51.976		25.13	.2144	25	19.542
	1.2881	1.2864		.925	.0007	4790	1.2877
	1.2886	1.2869		.1796	.2137	.920	.2206
3	48.936	48.226		35.98	.2505	25	19.542
	1.1867	1.1634		2.812	.0009	15880	1.1807
	1.1921	1.1690		.5954	.2496	1.188	.2613
4	49.833	46.916		47.04	.2077	25	19.542
	1.2161	1.1204		6.140	.0009	34170	1.1979
	1.2416	1.1507		1.2814	.2068	1.348	.3008
5	51.237	44.469		53.18	.2232	25	19.542
	1.2622	1.0401		8.691	.0008	47410	1.2223
	1.3111	1.1164		1.7777	.2224	1.389	.3162
6	52.551	41.608		56.69	.2280	25	19.542
	1.3053	.9463		10.395	.0008	55430	1.2502
	1.3722	1.0928		2.0787	.2272	1.396	.3251
7	38.695	37.736		33.73	.1780	25	19.542
	.8507	.8192		2.317	.0017	18330	.8427
	.8579	.8270		.6874	.1763	1.218	.1912

Table 5-18-continued

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted 1	Section 2	Depth (ft) 3	Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
8	39.546	30.955		45.70	.1704	25	19.542
	.8786	.5968		5.653	.0017	45330	.8314
	.9221	.7096		1.6999	.1687	1.387	.2147
9	43.564	39.357		45.45	.1880	25	19.542
	1.0104	.8724		5.565	.0012	37760	.9826
	1.0413	.9151		1.4159	.1868	1.369	.2505
10	34.831	34.680		22.91	.2106	25	19.542
	.7239	.7190		.683	.0022	6300	.7223
	.7248	.7198		.2363	.2084	.984	.1324
11	48.228	39.068		52.86	.2077	25	19.542
	1.1635	.8629		8.545	.0010	51120	1.1144
	1.2201	.9793		1.9170	.2067	1.392	.2890
12	52.416	52.361		25.18	.2434	25	19.542
	1.3009	1.2990		.931	.0007	4770	1.3004
	1.3014	1.2995		.1790	.2427	.9191	.2226
13	65.325	65.253		27.68	.4011	25	19.542
	1.7244	1.7220		1.260	.0004	4870	1.7237
	1.7249	1.7225		.1827	.4006	.924	.2966
14	63.809	62.358		46.82	.3046	25	19.542
	1.6746	1.6270		6.058	.0005	24270	1.6643
	1.6874	1.6405		.9101	.3042	1.277	.3957

Table 5-18-continued

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
15	63.626	62.199		46.74	.2996	25	19.542
	1.6687	1.6218		6.029	.0005	24230	1.6586
	1.6813	1.6353		.9087	.2992	1.276	.3943
16	65.895	64.574		46.84	.3140	25	19.542
	1.7431	1.6997		6.066	.0004	23320	1.7337
	1.7548	1.7121		.8747	.3136	1.268	.4096
17	68.210	65.175		53.63	.3610	25	19.542
	1.8190	1.7195		8.900	.0004	33040	1.7956
	1.8425	1.7459		1.2391	.3606	1.341	.4485
18	61.561	61.388		30.67	.4045	25	19.452
	1.6009	1.5952		1.734	.0005	7230	1.5992
	1.6020	1.5964		.2711	.4040	1.016	.3026

Table 5-19. Test Group 9; $\delta = .2908$ ft; $P_o = .0544$ ft²; $K = .7092$

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
1	39.067	38.387		31.30	.2120	25	19.542
	.8629	.8406		1.846	.0016	11190	.8567
	.8676	.8456		.5387	.2104	1.115	.2275
2	42.274	37.966		43.57	.2142	25	19.542
	.9681	.8268		4.928	.0014	27350	.9359
	.9959	.8662		1.3164	.2129	1.302	.2901
3	50.359	41.985		53.41	.2308	25	19.542
	1.2334	.9586		8.797	.0009	38490	1.1872
	1.2896	1.0601		1.8526	.2300	1.373	.3881
4	63.261	58.714		55.28	.3234	25	19.542
	1.6567	1.5075		9.689	.0005	30950	1.6256
	1.6930	1.5519		1.4900	.3229	1.327	.5139
5	64.607	62.746		47.60	.3683	25	19.542
	1.7008	1.6398		6.351	.0004	19560	1.6858
	1.7153	1.6553		.9418	.3679	1.232	.4944
6	62.467	61.847		37.90	.4226	25	19.542
	1.6306	1.6103		3.281	.0005	10490	1.6247
	1.6348	1.6145		.5048	.4222	1.102	.4261
7	37.184	36.910		25.97	.2263	25	19.542
	.8011	.7921		1.029	.0019	6690	.7983
	.8028	.7938		.3221	.2245	.998	.1897

Table 5-19-continued

Test No.	Cathetometer Reading (cm)			Point Gauge Reading (cm)	f'_c Combined	Temp. °C	Piezometer Spacing (ft)
	1	2	3				
	Contracted Section Depth (ft)			Discharge (ft ³ /sec)	f'_s Skin Friction	Reynolds Number	Spatial Mean Depth (ft)
	1	2	3				
	Flume Depth (ft)			Spatial Mean Velocity (ft/sec)	f'_D Drag	Drag Coefficient	f'_T
	1	2	3				
8	37.562	36.478		33.14	.1936	25	19.542
	.8135	.7780		2.197	.0018	14190	.8040
	.8211	.7863		.6830	.1918	1.165	.2230

oscillations were still present. Cathetometer readings given in the tables were obtained only for reasonably stable conditions. It will be noted that some of the readings are not converted into depths of flow. Only the readings used for calculations are converted in order to avoid confusion.

The combined friction factor f'_c is the friction factor determined by observation. The friction factor f'_s is the friction factor for skin friction and is determined using Equation 5-11. The difference between these two is the friction factor f'_D for the drag forces on the vertical obstructions. This friction factor is compared with the theoretical friction factor f'_T in Figure 5-4. The expression for the theoretical friction factor was given in Chapter III, and the Reynolds number and drag coefficient equations were given in Chapter II.

In Figure 5-4, experimentally determined friction factors f'_D are plotted against the theoretical friction factors f'_T . Experimental values are on the vertical axis and theoretical values are on the horizontal axis. Also in Figure 5-4 is a dashed line at 45 degrees. This line represents perfect agreement between theory and experiment. Although no distinction is made between the data points to indicate the variables involved, the plotted points fit the dashed line only in a rather general way. If velocity values are placed by each of the data points plotted, it becomes evident that in general the data points to the right of the dashed line correspond to higher velocities and data points to the left correspond to lower velocities. Thus higher velocities tend to reduce the actual friction factor to a value which is less than the theoretical value. This may be a consequence of sheltering which reduces the drag force on an obstruction or in effect its drag coefficient.

The theoretical equation from Chapter III is given here in a slightly different form as equation 5-6.

$$\frac{f'_T \delta}{d_m P_D} = C_D \quad 5-6$$

where f'_T = theoretical friction factor

δ	=	characteristic dimension
d_m	=	mean depth of flow
P_D	=	equivalent obstruction density
C_D	=	drag coefficient

It is obvious from Figure 5-4 that Equation 5-6 does not provide sufficient accuracy. There is some degree of agreement, but modifications must be made in order to improve the relationship. Velocity is certainly an important factor; however, velocity alone is not sufficient to account for the differences noted in Figure 5-4. It should be expected that the characteristic dimension is also a factor since the drag coefficients are dependent on the Reynolds number which contains the product of velocity and characteristic dimension. Obstruction density or the distribution of obstructions is also a factor, although this is not obvious from Figure 5-4. By substituting the experimental friction factor f'_D for the theoretical friction factor f'_T in Equation 5-6, the following Equation 5-7 is obtained.

$$\frac{f'_D \delta}{d_m P_D} = C_f = \eta C_D \quad 5-7$$

where f'_D = actual friction factor for vertical obstructions

$$\eta = \frac{C_f}{C_D} = \text{coefficient ratio (also equal to } f'_D/f'_T \text{)}$$

The term C_f will be called the friction coefficient. It will be assumed that C_f is directly proportional to C_D with the nature of this proportionality to be determined. Stated differently, it is assumed that an object's capability to produce energy losses is directly proportional to its capability to produce drag forces. How this proportionality is effected by the flow and obstruction parameters must be determined. C_f may also be considered as some percentage of C_D where this percentage may be greater than 100%. Perfect agreement between theory and experiment would make C_f equal to C_D , and the coefficient

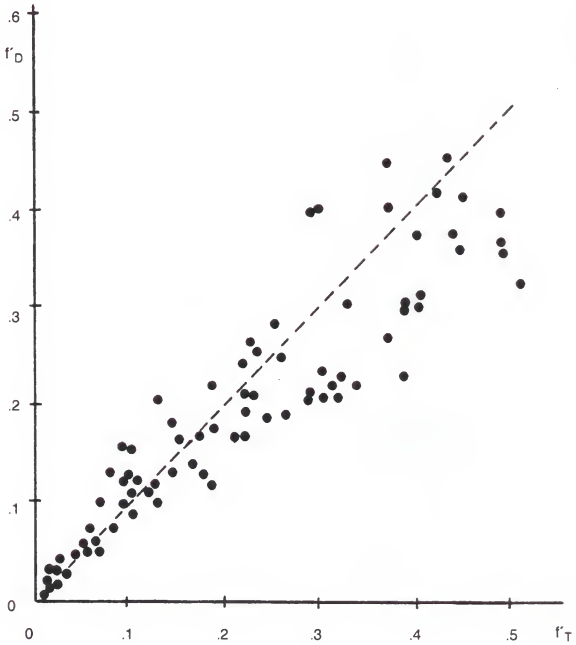


Figure 5-4: Experimental Friction Factor f'_D vs. Theoretical Friction Factor f'_T

ratio η would be 1.0, i.e., 100%. Dividing Equation 5-7 by C_D leads to the following Equation 5-8 where all terms have been defined.

$$\frac{f_D' \delta}{d_m P_D C_D} = \frac{C_f}{C_D} = \eta \quad 5-8$$

Values for the coefficient ratio in Equation 5-8 are tabulated with the corresponding mean velocities in Table 5-20. In all cases the coefficient ratio decreases as the velocity increases. Thus the proportionality between energy dissipation and drag forces is highly depended on velocity.

The data points in Table 5-20 are not plotted here, but there is a distinct variation in the coefficient ratio with obstruction density. This variation is not systematic. Thus some distinction must be made between the arrangements of vertical obstructions used to produce a given value of obstruction density. Specifically the spacing of the obstructions both laterally and longitudinally must be considered. The experiments done by Wang (1983) indicate that lateral spacing is of prime importance in determining an appropriate friction factor. The current experiments indicate that longitudinal spacing is important, although to a lesser degree. The current experiments alone, however, are not sufficient to investigate the variations in friction factors due to variations in the lateral spacing of obstructions. Therefore, information obtained from the Wang experiments will be used to examine lateral spacing variations. Other experimental results will also be considered.

Equivalent obstruction density is defined by Equation 5-9.

$$P_D = \frac{A_D}{A_L} = \frac{N \delta_m^2}{A_L} \quad 5-9$$

- where
- P_D = equivalent obstruction density
 - A_D = $N \delta_m^2$ = equivalent drag area
 - N = number of obstructions in area A_L
 - δ_m = mean value of characteristic dimension δ

Table 5-20. Coefficient Ratio η Versus Mean Velocity v_m

Test Group	η	v_m	Test Group	η	v_m
1	.9446	.9206	6	1.1485	.7876
	1.9406	.4055		.9543	1.2360
	1.6818	.5591		.7570	1.8649
2	1.4933	.4918		1.3081	.5675
	.9199	1.0350		1.2832	.5746
	1.3762	.6757		1.6093	.3185
3	1.0663	.6582	7	.9044	1.3677
	.6743	1.1768		1.1308	.8039
	1.2479	.3775		1.5857	.2933
4	.9926	.3748		1.7126	.3984
	.7253	.9009		1.4052	.5812
	.8648	.5666		1.0739	.7773
	.9144	.3586		1.0364	.8996
	.7190	.8721		.9564	1.0878
	.6424	1.2461		.9488	1.4543
	.8101	.6167		.9448	1.6315
5	1.0693	.7100	8	1.2700	.3233
	1.1644	.7386		.9685	.1796
	1.1162	.6373		.9553	.5954
	.7730	1.2008		.6876	1.2814
	.9223	1.0478		.7035	1.7777
	.9237	.7957		.6988	2.0787
	1.0747	.8891		.9221	.6874
	.8550	1.0687		.7857	1.6999
	.7440	1.2154		.7456	1.4159
	.7268	1.3355		1.5741	.2363
	.6554	1.5271		.7153	1.9170
	.6374	1.6258		1.0901	.1790
	1.2039	.3427		1.3507	.1827
	1.0482	.5577		.7686	.9101
	.9203	.7984		.7587	.9087
	.8132	1.0908		.7657	.8747
	.7197	1.3272		.8041	1.2391
	1.0818	.5873		1.3352	.2711
	.9866	.8669	9	.9248	.5387
	.7639	1.1874		.7339	1.3164
	.7114	1.3648		.5925	1.8526
				.6285	1.4900
				.7441	.9418
				.9907	.5048
				1.1833	.3221
				.8601	.6830

A_L = gross area under consideration (obstruction area plus flow area)

The convergence factor defined earlier to relate depths of flow in contracted and uncontracted sections of the laboratory flume is given here as Equation 5-10.

$$K = 1 - \frac{n\delta}{b} \quad 5-10$$

where K = convergence factor
 n = number of pipes per row
 δ = pipe diameter (characteristic dimension)
 b = flume width (width of unobstructed flow area)

The ratio term in Equation 5-10 may be used with a longitudinal spacing ratio to provide an alternative definition of P_D . The width of the unobstructed flow area divided by the number of obstructions per row is equal to the actual or average transverse spacing between centers of the obstructions. This transverse spacing will be denoted S_t and allows Equation 5-10 to be written as Equation 5-11. Note that for n obstructions per row there are actually $n + 1$ flow areas per row. It is assumed that the two outside flow areas are combined into a single flow area of width equal to the sum of the two individual widths. Thus n obstructions per row implies n flow areas per row when the two outer areas are combined, and S_t may be defined for one or any number of obstructions per row.

$$K = 1 - \frac{n\delta}{b} = 1 - \frac{1}{S_t/\delta} \quad 5-11$$

where $S_t = b/n$ = actual or average transverse spacing

Letting m denote the number of rows of obstructions per section in the laboratory flume or the number of rows in an area of flow length L , a dimensionless longitudinal spacing may be defined. This is given as Equation 5-12.

$$\frac{S_l}{\delta} = \frac{L}{m\delta} \quad 5-12$$

where S_l = longitudinal spacing

$\frac{S_l}{\delta}$ = dimensionless longitudinal spacing

L = length of flow area

m = number of rows of obstructions in length L

For the tests conducted in the laboratory flume the mean characteristic dimension was equal to the pipe diameter used for the test considered. In all tests but those of groups 5 and 6 the pipes were arranged in systematic transverse and longitudinal rows, and the total number of obstructions was equal to n times m . Since the total area was equal to b times L , apparent obstruction density may also be expressed by the following Equation 5-13 where all terms have been defined.

$$P_D = \frac{N\delta_m^2}{A_L} = \frac{nm\delta^2}{bL} = \frac{\delta^2}{S_t S_l} = \frac{1}{\frac{S_t}{\delta} \cdot \frac{S_l}{\delta}} \quad 5-13$$

The staggered arrangement of pipes in groups 5 and 6 will be considered later in this chapter; however, the alternative definition of P_D may be used in Equation 5-8 to provide Equation 5-14. All terms have been defined.

$$f'_D = \frac{\eta C_D \frac{d_m}{\delta}}{\frac{S_t}{\delta} \cdot \frac{S_l}{\delta}} \quad 5-14$$

With the exception of η which will be shown to have dimensional terms, Equation 5-14 implies that the friction factor is geometrically dependent on four dimensionless terms. Thus the scale of the system is important only with regard to η . The variation of η will be considered in the following paragraphs. Note that it is essential to include velocity in this analysis.

Using mean depth as hydraulic radius, the Darcy-Weisbach equation may be expressed as the following Equation 5-15.

$$\Delta H_f = f' \cdot \frac{v_m^2}{2g} \cdot \frac{L}{d_m} \quad 5-15$$

where ΔH_f = head loss due to friction
 f' = friction factor based on hydraulic radius
 v_m = mean velocity
 g = acceleration due to gravity
 L = length of flow field
 d_m = mean depth of flow

Combining Equations 5-14 and 5-15 by substituting the friction factor due to vertical obstructions, an expression for the energy loss due to the vertical obstructions is obtained. This is given as Equation 5-16.

$$\Delta H_v = \frac{\eta C_D \frac{d_m}{\delta}}{\frac{S_t}{\delta} \cdot \frac{S_l}{\delta}} \cdot \frac{v_m^2}{2g} \cdot \frac{L}{d_m} \quad 5-16$$

where ΔH_v = head loss due to vertical obstructions

Equation 5-16 may be expressed in terms of head loss per row by dividing through by the number of rows. It may be made nondimensional by dividing through by δ . Also dividing through by the drag coefficient as was done previously, a dimensionless head loss per row may be expressed by the following Equation 5-17

$$\frac{\Delta H_v}{m C_D} = \frac{\eta}{S_t} \cdot \frac{v_m^2}{2g} = \frac{f_D' \delta}{d_m P_D C_D} \cdot \frac{v_m^2}{2g S_t} \quad 5-17$$

For each of the laboratory tests the natural logarithm of the dimensionless expression on the right-hand side of Equation 5-17 is evaluated and tabulated with the natural logarithm of the corresponding Reynolds number in Table 5-21. These data points are

separated by test group and are plotted in Figures 5-5 and 5-6 for which the test group numbers are indicated. In every case the relationship between the logarithms is linear, although a few data points appear to be in error. In general, however, the linear relationship is excellent. It is also apparent from Figures 5-5 and 5-6 that straight lines representing each test group all have approximately the same slope. Based on test groups 4 through 9 a common slope of 1.619 is assumed for all test groups. All of the lines drawn in Figures 5-5 and 5-6 have this slope. Using the common slope of 1.619 the relationship between the variables may be expressed by Equation 5-18.

$$\ln \left[\frac{f_D \delta}{d_m P_D C_D} \cdot \frac{v_m^2}{2g S_t} \right] = 1.619 \ln R\theta_\delta - H_c \quad 5-18$$

where H_c = spacing function for circular cylinders

For the laboratory tests with circular cylinders, the spacing function introduced by Equation 5-18 should depend only on the geometric properties of the obstructions; namely, characteristic dimension and the dimensionless spacings. The most representative numerical values of H_c for all test groups are given in Table 5-22. These values are obtained from the data in Table 5-21 for which data points considered to be in error are denoted by an asterisk and are omitted in the determination of H_c values. Also included in Table 5-22 are the corresponding values of S_t/δ , S_t/δ , and δ .

In Figure 5-7 the values of H_c from Table 5-22 are plotted against S_t/δ for all test groups. This plot is suggested by the H_c values for groups 1 through 4 for which S_t/δ and δ are constant. The rate of change of H_c indicated by groups 1 through 4 may be approximated by a power expression or by a logarithmic expression for the small range of spacing values considered. A logarithmic expression is better suited to Equation 5-18; however, a power expression will be used here. Note that neither of these expressions would have an upper limit to the effect of spacing on the value of H_c . Physically of course

Table 5-21. $\ln \left[\frac{f'_0 \delta}{d_m P_0 C_0} \cdot \frac{v_m^2}{2gS_t} \right]$ vs $\ln (Re_\delta)$

Test Group	$\ln(\eta v_m^2/2gS_t)$	$\ln (Re_\delta)$	Test Group	$\ln(\eta v_m^2/2gS_t)$	$\ln (Re_\delta)$
1	-4.6743	9.4632*	6	-4.7909	9.3072
	-5.5939	8.6434		-4.0748	9.7578
	-5.0947	8.9646		-3.4838	10.1691*
2	-5.4701	8.8363		-5.3163	8.9794
	-4.4666	9.5803		-5.3106	8.9919
	-4.9165	9.1539		-6.2641	8.4019
3	-5.2241	9.1277	7	-3.9262	10.5043
	-4.5203	9.7088*		-4.7655	9.9729
	-6.1789	8.5717		-6.4442	8.9645
4	-6.4217	8.5647		-5.7545	9.2709
	-4.9818	9.4415		-5.1972	9.6485
	-5.7332	8.9779		-4.8846	9.9392
	-6.5927	8.5203*		-4.6278	10.0854
	-5.0554	9.4091		-4.3282	10.2753
	-4.4542	9.7660		-3.7555	10.5657
5	-5.6291	9.0626		-3.5298	10.6807
	-5.0697	9.2035	8	-6.4712	9.0621
	-4.9057	9.2429*		-7.9179	8.4742*
	-5.2428	9.0955		-5.5347	9.6727
	-4.3432	9.7290		-4.3307	10.4391
	-4.4393	9.5927		-3.6530	10.7665
	-4.9882	9.3174		-3.3469	10.9229
	-4.6149	9.4284*		-5.2829	9.8163
	-4.4756	9.6124		-3.6320	10.7217
	-4.3573	9.7410		-4.0501	10.5389
	-4.1924	9.8352		-6.8832	8.7487
	-4.0275	9.9693		-3.4855	10.8419
	-3.9301	10.0319		-7.8064	8.4708*
	-6.4081	8.4750		-7.5509	8.4914
	-5.5725	8.9620		-4.9036	10.0969
	-4.9852	9.3207		-4.9195	10.0954
	-4.4848	9.6329		-4.9868	10.0573
	-4.2145	9.8290		-4.2412	10.4056
	-5.4375	9.0138		-6.7734	8.8859
	-4.7509	9.4031	9	-5.4799	9.3228
	-4.3776	9.7177		-3.9240	10.2163*
	-4.1703	9.8569		-3.4546	10.5580
				-3.8313	10.3402
				4.5799	9.8815
				-5.5409	9.2579
				-6.2619	8.8085
				-5.0775	9.5602

*value considered to be in error

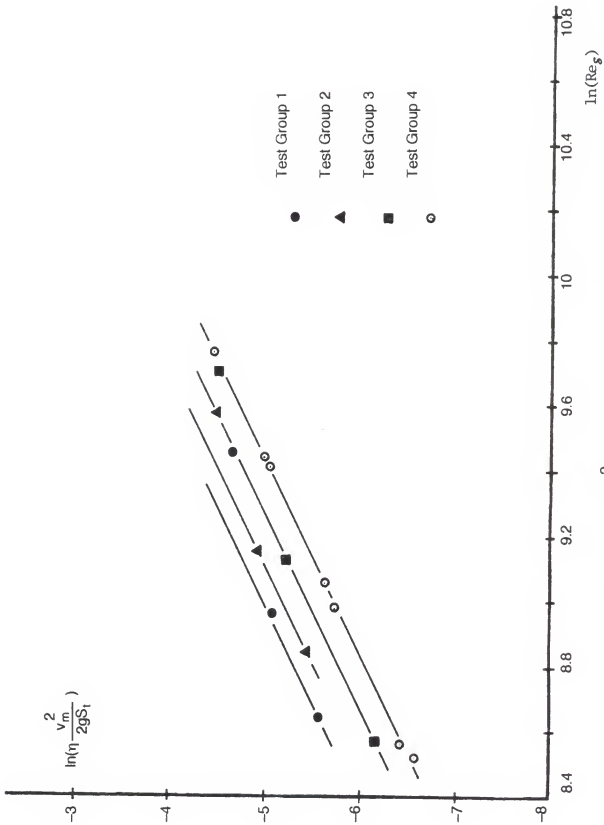


Figure 5-5. $\ln(\eta \frac{v_m^2}{2gS_1})$ vs. $\ln(Re_s)$, Test Groups 1-4

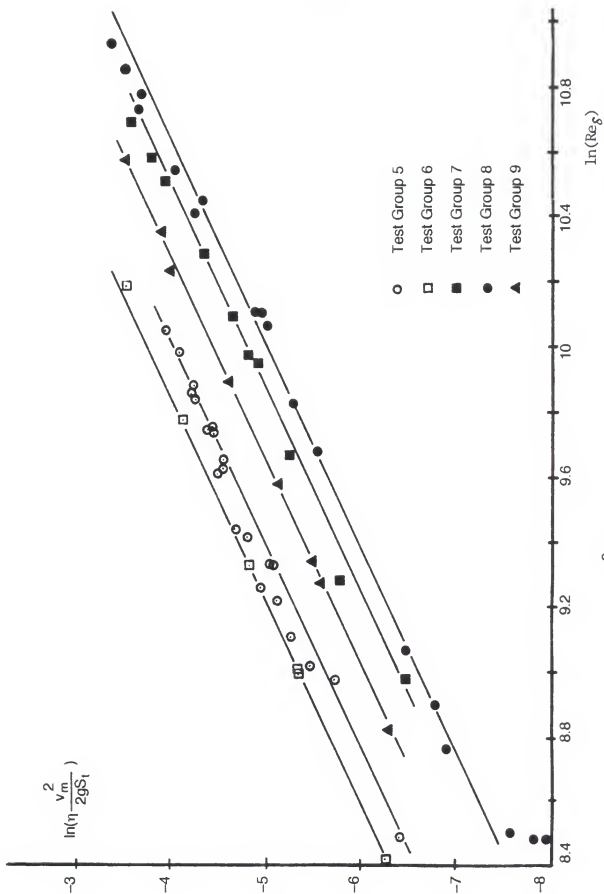


Figure 5-6. $\ln(\eta \frac{v_m^2}{2gS_t})$ vs. $\ln(Re_\Delta)$, Test Groups 5-9

Table 5-22. Spacing Function H_c and Geometric Factors

Test Group	H_c	S_t/δ	S_b/δ	δ (ft)
1	19.598	6.809	49.895	.1958
2	19.830	6.809	24.949	.1958
3	20.029	6.809	12.473	.1958
4	20.280	6.809	6.238	.1958
5	20.076	6.809	6.238	.1958
6	19.864	6.809	12.473	.1958
7	20.896	3.571	13.087	.3733
8	21.149	3.571	4.027	.3733
9	20.554	3.438	4.201	.2908

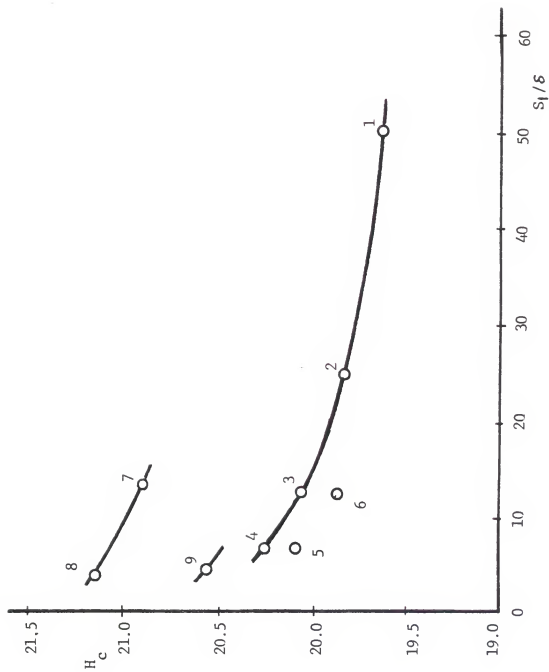


Figure 5-7: Spacing Function H_c vs. Dimensionless Longitudinal Spacing S_l/ϵ

there is an upper limit to the longitudinal spacing of obstructions for which an upstream obstruction effects another downstream.

Like groups 1 through 4 the values of δ and S_l/δ are constant for test groups 7 and 8. Although the number of data points in Figure 5-7 is limited, the rate of change of H_c with S_l/δ appears to be approximately the same for groups 7 and 8 as for groups 1 through 4. If the rates are expressed in terms of power formulas, however, the rates are seen to be different. This difference is assumed to depend on the scale of the system, i.e., on δ . Groups 1 through 4 and groups 7 and 8 may be represented by Equation 5-19. Note that H_1 is different for each set of data points. $H_1 = 20.894$ for groups 1 through 4, and $H_1 = 21.451$ for groups 7 and 8.

$$H_c = \frac{H_1}{\left(\frac{S_l}{\delta}\right)^{0.0049/(\delta^{0.743})}} \quad 5-19$$

where H_c = spacing function for circular cylinders
 H_1 = fractional spacing function
 S_l/δ = dimensionless longitudinal spacing
 δ = characteristic dimension in feet

Equation 5-19 may be substituted into Equation 5-18 to yield the following Equation 5-20.

$$\ln \left[\frac{f_D' \delta}{d_m P_D C_D} \cdot \frac{v_m^2}{2gS_t} \right] = 1.619 \ln Re_\delta - \frac{H_1}{\left(\frac{S_l}{\delta}\right)^{0.0049/(\delta^{0.743})}} \quad 5-20$$

where δ = characteristic dimension in ft

It should be noted explicitly that Equation 5-20 is not dimensionless. The scale of the system is now included through the characteristic dimension in the exponent of S_l/δ . Note also that this exponent approaches zero for large δ values. This implies that for large characteristic dimensions the dimensionless longitudinal spacing has little effect on the

head loss per obstruction. The form of Equation 5-19 is based on the observation that in all tests the value of H_c decreased when S_f/δ was increased. Decreasing H_c implies a larger head loss which is consistent with increased spacing and reduced sheltering. Equation 5-19 satisfies this observation for all possible values of S_f/δ and δ . The rate of change indicated by Equation 5-19 is based on the experimental data. Each line drawn through the data points in Figure 5-7 is a plot of Equation 5-19. The rate of change indicated by the equation is assumed to include the combined effects of sheltering of the obstructions and the decay of turbulence as the longitudinal spacing is increased. It is doubtful that these two effects may be considered separately.

The fractional spacing function in Equation 5-20 is independent of S_f/δ but varies with δ and S_f/δ . It is constant for a given characteristic dimension and transverse spacing. How H_c or H_1 varies with δ or S_f/δ cannot be determined with only the limited information available from the current experiments. The basic form of the relationship between H_c or H_1 and δ or S_f/δ may however be determined through the use of experimental results from other investigations. The experiments performed by Wang (1983), by Nagler (1918), by Yarnell (1934), and by d'Aubuisson (see Chow, 1959) indicate that H_c or H_1 varies as a function of $((S_f/\delta) - 1.0)$ and that the rate of change with respect to δ is logarithmic in form. In some of the tests a power formula best represents the $((S_f/\delta) - 1.0)$ function, and in others a logarithmic expression is better. For the limited range of S_f/δ values normally encountered either representation is sufficient. A power formula will be used here to express the rate of change with $((S_f/\delta) - 1.0)$, and a logarithmic expression will be used to represent the variations with characteristic dimension δ . It should be noted that 1.0 is the minimum possible value of S_f/δ for obstructions of equal characteristic dimension.

Values of H_1 are computed for all test groups and are given in Table 5-23. Also included in the table are the corresponding values of S_f/δ and δ . It should be noted that test groups 4 and 5 have equal values of δ , S_f/δ , and S_f/δ ; however, group 5 has twice the equivalent obstruction density of group 4. Likewise, groups 3 and 6 have common

Table 5-23. Fractional Spacing Function H_f and Parameters S_f/δ and δ

Test Group	H_f	S_f/δ	δ (ft)
1	20.901	6.809	.1958
2	20.908	6.809	.1958
3	20.878	6.809	.1958
4	20.900	6.809	.1958
5	20.690	6.809	.1958
6	20.706	6.809	.1958
7	21.451	3.571	.3733
8	21.451	3.571	.3733
9	20.919	3.438	.2908

values of δ , S_t/δ , and S_t/δ , but group 6 has twice the density of group 3. If the ratio f'_D/P_D is the same for groups 4 and 5 and for groups 3 and 6, then the assumption that f'_D is proportional to P_D is a good one. Comparison of these two sets of data indicates an increase in the ratio of approximately 18% for the higher densities. This increase is not due to an error in the assumption but is considered to be due to distortion of the flow pattern between adjacent obstructions caused by the offset obstructions upstream. This distortion causes slightly higher velocities approaching the obstructions and an increase in head loss. Groups 5 and 6 with the offset obstructions have the higher ratio and thus the greatest loss of energy. The proportionality between f'_D and P_D is considered to be excellent. The small difference in the values of H_1 for the groups mentioned above may be seen in Table 5-23. The values from Table 5-23 and the basic forms representing the rates of change with δ and S_t/δ may be used to develop Equation 5-21. This equation will be compared with similar equations based on the experimental results of other investigations.

$$H_1 = (22.46 + 1.87 \ln \delta) \left[\frac{S_t}{\delta} - 1.0 \right]^{0.042} \quad 5-21$$

Equation 5-21 may be substituted into Equation 5-20 to give Equation 5-22.

$$\ln \left[\frac{f'_D \delta}{d_m P_D C_D} \cdot \frac{v_m^2}{2g S_t} \right] = 1.619 \ln Re_\delta - \frac{(22.46 + 1.87 \ln \delta) \left[\frac{S_t}{\delta} - 1.0 \right]^{0.042}}{\left(\frac{S_t}{\delta} \right)^{0.0049/(\delta^{0.743})}} \quad 5-22$$

Letting the ratio of power expressions in Equation 5-22 be denoted by ω , the equation may be expressed in the following form, Equation 5-23.

$$\frac{f'_D \delta}{d_m P_D C_D} \cdot \frac{v_m^2}{2g S_t} = \frac{1.619}{\left(\frac{22.46 + 1.87}{\delta^{0.743}} \right) \omega} \quad 5-23$$

where

$$\omega = \frac{((S_f/\delta) - 1.0)^{0.042}}{\left(\frac{S_f}{\delta}\right)^{0.0049/(\delta^{0.743})}}$$

$$e = 2.71828 \dots = \text{base for natural logarithm}$$

Recalling the definition of η , Equation 5-23 may be solved for η . This is given as Equation 5-24. Multiplying Equation 5-24 by C_D gives C_f , the friction coefficient.

$$\eta = \frac{2g S_f Re_\delta^{1.619}}{v_m^2 \left(e^{22.46} \delta^{1.87} \right)^\omega} \quad 5-24$$

where $\eta = C_f / C_D = \text{coefficient ratio}$

Equation 5-23 may also be solved for f'_D . This is given as Equation 5-25.

$$f'_D = \frac{2g S_f Re_\delta^{1.619}}{v_m^2 \left(e^{22.46} \delta^{1.87} \right)^\omega} C_D \frac{d_m}{\delta} P_D \quad 5-25$$

Note that Equation 5-25 has the same basic form as the theoretical expression developed earlier in Chapter III. The distribution factor defined earlier is not included for the equal diameter obstructions. The coefficient of $C_D(d_m/\delta)P_D$ represents the modifications necessary to account for velocity, characteristic dimension, and the spacing of the obstructions.

The friction factor f'_D from Equation 5-25 may be substituted into the Darcy-Weisbach equation from which the following Equation 5-26 may be developed for head loss per row.

$$\frac{\Delta H}{m} = \frac{Re_\delta^{1.619}}{\left(e^{22.46} \delta^{1.87} \right)^\omega} C_D \delta \quad 5-26$$

about 1.0 which implies that head losses are proportional to δ to about the 0.75 power. Note specifically that head losses are directly proportional to the drag coefficient C_D . Thus it is necessary to use the most accurate drag coefficients available. In actual practice the determination of a suitable drag coefficient is largely a matter of estimation. Obstruction shapes may vary widely, and the direction of flow may not match some predetermined direction for which an appropriate coefficient is known. It would be desirable to express the present data in a form which does not depend on the drag coefficient. This may be done by modifying Equation 5-18 given previously. Eliminating C_D from that equation leads to Equation 5-27.

$$\ln \left[\frac{f_D' \delta}{d_m P_D} \frac{v_m^2}{2gS_1} \right] = 1.782 \ln Re_\delta - \frac{(23.951 + 1.943 \ln \delta) \left[\frac{S_1}{\delta} - 1.0 \right]^{0.040}}{\left(\frac{S_1}{\delta} \right)^{0.00543/\delta} (\delta^{0.76203})} \quad 5-27$$

Equation 5-27 may be put into the same form as Equation 5-26. This is given as Equation 5-28.

$$\frac{\Delta H}{m} = \frac{Re_\delta^{1.782} \delta}{(e^{23.951} \delta^{1.943}) \omega'} \quad 5-28$$

where

$$\omega' = \frac{((S_1/\delta) - 1.0)^{0.040}}{\left(\frac{S_1}{\delta} \right)^{0.00543/\delta} (\delta^{0.76203})}$$

Although Equations 5-27 and 5-28 are included here, they are primarily for comparison. Equations 5-22 through 5-26 with variable drag coefficients are preferable. These latter equations will be compared with some of the experimental results from other sources as mentioned earlier. This is considered in the following paragraphs.

Wang determined head losses for rows of concrete blocks in the same flume used for the current experiments. One set of those tests considered a single row of blocks where

Wang determined head losses for rows of concrete blocks in the same flume used for the current experiments. One set of those tests considered a single row of blocks where the number of blocks per row was varied from two to ten. Table 5-24 is derived from data presented by Wang (1983), and the dimensionless transverse spacing is computed in accordance with Equation 5-11. Note that metric units are used for this data. The flume width $b = 8.00 \text{ ft} = 2.4384 \text{ meters}$, and the characteristic dimension $\delta = 7.5 \text{ inches} = 0.1905 \text{ meters}$.

For each block pattern indicated in Table 5-24 the range of velocities considered for the indicated number of tests was relatively small. The term $\overline{v_m^2}$ represents the mean of the squares of these individual velocities. Note that v_m^2 is the quantity required by Equation 5-18. The square root of $\overline{v_m^2}$ is used for determination of Reynolds number. The term $\overline{\Delta H}$ indicates the average head loss for each test pattern. The term in the next-to-last column of Table 5-24 is equivalent to the coefficient ratio in Equation 5-8. This may be ascertained by the relationship given in Equation 5-17. The acceleration due to gravity is set equal to 9.807 ms^{-2} , C_D is estimated at 2.0 for the rectangular concrete blocks, and the number of rows is equal to 1.0. Note that for a single row of blocks the longitudinal spacing is undefined. An appropriate S_l value would be the longitudinal spacing of multiple rows of blocks for which the upstream turbulence acting on any individual row would match the turbulence generated in the test area for the single row. An estimate of this length would be about 50 ft, the distance between the blocks and the upstream end of the flume. Note in Equation 5-20 that the longitudinal spacing affects the numerical value of the spacing function. Numerical values of H_c as required by Equation 5-18 and given in the last column of Table 5-24. These values are the same order of magnitude as the values given previously in Table 5-22.

The values of H_c given in Table 5-24 may be plotted against values of $((S_l/\delta) - 1.0)$ to indicate the relationship between the variables. Although a plot of the data points is not provided here, the following Equation 5-29 is an excellent fit.

Table 5-24. Average Experimental Values – Wang

Pattern	Number of Blocks per Row	Number of Tests	$\overline{\Delta H}$ (m)	$\overline{v_m^2}$ (m ² /s ²)	S_t/δ	$\frac{2g \overline{\Delta H} (S_t/\delta)}{mC_o v_m^2}$	H
1	2	12	.0019	.2666	6.400	.452	23.478
2	4	10	.0075	.2119	3.200	1.115	21.925
3	6	10	.0163	.1760	2.133	1.937	21.003
4	8	10	.0393	.1500	1.600	4.115	19.992
5	10	17	.0639	.0768	1.280	10.436	18.966

$$H_b = 20.7702 \left[\frac{S_t}{\delta} - 1.0 \right]^{0.072} \quad 5-29$$

where H_b = spacing function for concrete blocks tested

S_t/δ = dimensionless transverse spacing

Note that the exponent in Equation 5-29 has a value of 0.072. This is substantially different from the exponent 0.042 in Equation 5-21. Some of this difference is due to the assumption that the drag coefficient is constant at 2.0 for the concrete blocks. There is probably some variation which affects the exponent since the velocities considered during the tests varied by a factor of about 2. Other exponents will be considered in the following paragraphs. Using a longitudinal spacing value of 50 ft and a characteristic dimension of 0.625 ft, the coefficient in Equation 5-29 may be calculated by Equation 5-22. Equation 5-22 gives a value of 20.93. This is reasonably close to 20.77 as in Equation 5-29.

Chow (1959) and Henderson (1966) have considered the results of numerous experiments concerned with the flow of water through bridge piers. These flows are similar to the flow between vertical obstructions in a coastal flooding situation. It is therefore of interest to compare the bridge pier results with the current equations.

Chow presents a formula due to d'Aubuisson which was apparently developed approximately 150 years ago. This formula is given as the following Equation 5-30 for which a single vertical obstruction is placed in the center of the flow area. Section 1 refers to a section just upstream of the obstruction, section 3 refers to a section just downstream, and section 2 refers to a section passing through the obstruction and the contracted flow. This designation of sections will be used for all of the following equations.

$$Q = K_A b_2 Y_3 \sqrt{2gh_3 + v_1^2} \quad 5-30$$

where Q = discharge

- K_A = d'Aubuisson's coefficient
 b_2 = width of the flow area in the obstructed section 2
 Y_3 = depth of flow in the downstream section 3
 g = acceleration due to gravity
 h_3 = $Y_1 - Y_3$ = difference in piezometric head between sections 1 and 3
 Y_1 = depth of flow in upstream section 1
 v_1 = mean velocity in upstream section 1

Chow also presents an improved formula due to Nagler (1918) which is given as the following Equation 5-31. It is similar in form to Equation 5-30 but contains additional coefficients. Nagler also considered multiple obstructions in the same row.

$$Q = K_N b_2 \sqrt{2g} \left[Y_3 - \theta \frac{v_3^2}{2g} \right] \sqrt{h_3 + \beta \frac{v_1^2}{2g}} \quad 5-31$$

- where K_N = Nagler's coefficient
 θ = adjustment factor
 v_3 = mean velocity in downstream section 3
 β = velocity coefficient (this is not α as in the energy equation)

Chow defines a conveyance ratio as the width of the flow in the contracted section divided by the width of the unobstructed channel. This ratio is equal to the convergence factor defined previously. If all obstructions have the same characteristic dimension, the following Equation 5-32 may be used. This equation is valid for any number of obstructions and any spacing. Recall that the two outer flow areas are considered to be combined into a single flow area.

$$\sigma = K = 1 - \frac{1}{S_t/b} \quad 5-32$$

where	σ	=	conveyance ratio = b_2/B
	K	=	convergence factor
	S_1	=	actual or average transverse spacing
	δ	=	characteristic dimension
	b_2	=	total flow width in obstructed section
	B	=	unobstructed flow width

Henderson gives an empirical equation due to Yarnell (1934) which is given here as Equation 5-33.

$$\frac{Y_1 - Y_3}{Y_3} = K_Y Fr_3^2 (K_Y + 5 Fr_3^2 - 0.6) (\alpha + 15\alpha^4) \quad 5-33$$

where	K_Y	=	Yarnell's coefficient
	Fr_3	=	$v_3 / \sqrt{gY_3}$ = Froude number in downstream section 3
	α	=	$1 - \sigma$ = ratio of pier width to unobstructed channel width

Letting the entire right hand side of Equation 5-33 be denoted by ϕ_Y , the following Equations 5-34 may be developed.

$$\frac{Y_1 - Y_3}{Y_3} = \phi_Y \quad 5-34A$$

$$Y_1 - Y_3 = \phi_Y Y_3 \quad 5-34B$$

$$\frac{Y_1}{Y_3} = 1 + \phi_Y \quad 5-34C$$

where	ϕ_Y	=	phi function for Yarnell equation
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Applying the equation of continuity between sections 1 and 3 will give the following Equation 5-35.

$$Fr_3^2 = (1 + \phi_Y)^3 Fr_1^2 \quad 5-35$$

Applying the energy equation between sections 1 and 3 gives Equation 5-36. Note that velocity heads have now been included in the determination of losses.

$$\frac{\Delta H}{Y_1} = \frac{\phi_Y}{1 + \phi_Y} - Fr_1^2 \left[\phi_Y + \frac{\phi_Y^2}{2} \right] \quad 5-36$$

where ΔH = total energy loss between sections 1 and 3.

The Nagler and d'Aubuisson equations given previously may also be put into the form of Equation 5-34A. This is given as Equation 5-37.

$$\frac{Y_1 - Y_3}{Y_3} = \frac{Fr_3^2 + 2K_B^2 \sigma^2 \left[1 - \frac{\theta}{2} Fr_3^2 \right]^2}{2K_B^2 \sigma^2 \left[1 - \frac{\theta}{2} Fr_3^2 \right]^2 \left[1 + \frac{\beta}{2} Fr_1^2 \right]} - 1 \quad 5-37$$

where K_B = Nagler's coefficient K_N or d'Aubuisson's coefficient K_A

σ = $1 - \alpha = 1 - \frac{1}{S_d \delta} = K$ = convergence factor

θ = Nagler's adjustment factor

θ = zero in d'Aubuisson formula

β = Nagler's velocity coefficient

β = 1 in d'Aubuisson formula

If the entire right-hand side of Equation 5-37 is denoted as ϕ_N for Nagler's coefficients or as ϕ_A for d'Aubuisson's coefficients, then Equation 5-37 may be written as the following Equation 5-38. This is the same as Equation 5-36 except for the designation of the ϕ function.

$$\frac{\Delta H}{Y_1} = \frac{\phi}{1 + \phi} - Fr_1^2 \left[\phi + \frac{\phi^2}{2} \right] \quad 5-38$$

where ϕ = Nagler's ϕ function from Equation 5-37

or ϕ = d'Aubuisson's ϕ function from Equation 5-37

or $\phi =$ Yarnell's ϕ function from Equation 5-33

Introducing the Darcy-Weisbach equation into Equation 5-38 allows the equation to be written in terms of the dimensionless ratio of variables used previously. This is given as the following Equation 5-39.

$$\frac{f_D' \delta}{d_m P_D C_D} \cdot \frac{v_m^2}{2g S_t} = \frac{Y_1}{m \delta C_D} \left[\frac{\phi}{1 + \phi} - Fr_1^2 \left[\phi + \frac{\phi^2}{2} \right] \right] \quad 5-39$$

Thus the dimensionless ratio of variables used in the current development may be determined for each of the three experimental relationships given. In all cases the ϕ function is iterative and of course the appropriate coefficients must be used. The Yarnell and d'Aubuisson equations have a single coefficient, K_Y and K_A , respectively. The Nagler equation, however, has three coefficients - K_N , θ , and β . K_N is shape dependent like K_Y and K_A . According to Chow, the θ value varies, approaching zero as the contraction effect approaches zero. It becomes high when the flow is shooting and turbulent. Apparently this implies low σ values and/or high velocities. Chow suggests a value of 0.3 for ordinary use. This value will be used here with Nagler's equation. Note that the θ value ordinarily has only a minor affect on the results. The velocity coefficient β in Nagler's equation is not the normal velocity head coefficient used with the energy equation. It is a factor used to correct for the velocity of approach. Both Chow and Nagler present β graphically as a function of σ ; however, the following Equation 5-40 may be used to determine β . The results of this equation are very close to the graphical results.

$$\beta = 2.077 - 3.0 \cdot 10^{-4} e^{9.2\sigma} + 5.8 \cdot 10^{-7} e^{15.0\sigma} \quad 5-40$$

where $\beta =$ Nagler's velocity correction coefficient
 $\sigma =$ conveyance ratio
 $e = 2.71828 \dots =$ base for natural logarithm

The following Table 5-25 gives the values of K_A , K_N , and K_Y as a function of the shape of the piers and the conveyance ratio σ . Note that Yarnell's coefficients are not given as a function of σ although he considered σ values of 0.993, 0.767, 0.650, and 0.500. Apparently the form of his equation is such that a single coefficient applies to all of the σ values he considered.

The tests performed by Wang considered a single δ value of 0.625 ft and varied σ by altering the number of obstructions. The depths considered varied from approximately 0.45 to 0.75 ft. Nagler considered a single δ value of 0.5 ft and depths in the range of 2 to 3 ft. Yarnell considered depths in the 1 to 2.5 ft range, and three different δ values were used – 14 inches, 42 inches, and 60 inches. d'Aubuisson's test conditions are not known. In Equation 5-22 given previously, the ratio of power expressions was denoted ω . This ratio has a coefficient which is dependent on δ . Therefore to compare the various test results it is necessary to use a common δ value. A value of 0.625 ft is selected to match the Wang data. It is assumed that the other experimental equations are valid for δ values other than those specifically tested. A common depth of 2.0 ft will also be used. Equation 5-22 may be expressed as the following Equation 5-41.

$$\ln \left[\frac{f_D' \delta}{d_m P_D C_d} \cdot \frac{v_m^2}{2g S_t} \right] = A \ln Re_\delta - B \left[\frac{S_t}{\delta} - 1.0 \right]^C \quad 5-41$$

where

A	=	exponent of the Reynolds number in the head loss per row expression
B	=	coefficient of the transverse spacing power expression
C	=	exponent in the transverse spacing power expression

It should be noted that the coefficient B is dependent on δ and also on the longitudinal spacing. In Table 5-26 the values of A , B , and C are given for some of the experimental equations and pier shapes. An assumed drag coefficient is also included in the table and a comment is given regarding how well Equation 5-41 fits the experimental results for the bridge piers. Correlation coefficients are also given in the table, and σ values of 0.5, 0.6,

Table 5-25. Contraction Coefficients K_A , K_N , and K_V

Type of Pier	Conveyance Ratio α											
	.9			.8			.7			.6		
	K_A	K_N	K_V	K_A	K_N	K_V	K_A	K_N	K_V	K_A	K_N	K_V
Square Nose and Tail	.96	.91	1.25	1.02	.87	1.25	1.02	.86	1.25	1.00	.87	1.25
Semi-Circular Nose and Tail	.99	.94	.90	1.13	.92	.90	1.20	.95	.90	1.26	1.03	.90
90° Triangular Nose and Tail	—	.95	1.05	—	.94	1.05	—	.92	1.05	—	—	1.05
Lens-shaped Nose and Tail	1.00	.95	.90	1.14	.94	.90	1.22	.97	.90	—	—	.90
Twin Cylinder with Connecting Diaphragm	—	.91	.95	—	.89	.95	—	.88	.95	—	—	.95
Twin Cylinder without Connecting Diaphragm	—	.91	1.05	—	.89	1.05	—	.88	1.05	—	—	1.05

Table 5-26. Numerical Values of A, B, and C for Equation 5-41 ($\delta = 0.625$ feet, $Y_1 = 2.0$ feet)

Equation	Shape	A	B	C	C_0	(Correlation Coefficient)	$\frac{\Delta H}{m}$ (ft)
Current	Circular Cylinder	1.619	20.935*	0.042	Eq 2-7 → 2.9	Exact Fit (.10000)	.0184
Wang and Christensen	Concrete Block Parallel to Flow	1.619 Assumed	20.7702	0.072	2.0	Excellent Fit (.9997)	.0139
Nagler	Square Nose and Tail	2.0203	25.0130	0.0487	2.0	Good Fit (.9940)	.0180
Yarnell	Square Nose and Tail	2.0447	25.5155	0.0427	2.0	Good Fit (.9961)	.0155
d'Aubuisson	Square Nose and Tail	2.0086	24.9498	0.0376	2.0	Fair Fit (.9775)	.0206
Wang and Christensen	Concrete Block Parallel to Flow	2.0245 Assumed	25.2170	0.0626	2.0	Excellent Fit (.9997)	.0118
Nagler	Semi-circular Nose and Tail	2.0230	25.6226	0.0484	1.5	Fair Fit (.9883)	.0074
Nagler	Twin Cylinder without Diaphragm	2.0178	25.8561	0.0376	1.3	Poor Fit (.9428)	.0058
Yarnell	Semi-circular Nose and Tail	2.0852	26.6405	0.0422	1.5	Good Fit (.9949)	.0057
Yarnell	Twin Cylinder without Diaphragm	2.0596	25.7322	0.0432	1.3	Good Fit (.9954)	.0094

*Assumes longitudinal spacing of 50.0 feet

0.7, 0.8 and 0.9 are used in all cases except those for which appropriate coefficients are not available. A range of Reynolds numbers is used which corresponds to the Reynolds numbers of the current experiments and also the Wang experiments. In general, velocities of 0.1, 0.5, and 1.0 fps are used. Higher velocities cause a slight curvature to the linear relationship given previously as Equation 5-18. Note that all of the experimental ϕ functions become undefined for high velocities. Chow notes that neither the Nagler nor the d'Aubuisson formulas apply too well at high velocities. Yarnell considered high velocities and provided graphical solutions for these conditions. They are not considered directly by his equation.

The exponent A in Table 5-26 defines the slope of the lines given in the previous Figures 5-5 and 5-6. For the circular obstructions C_D is a function of v_m , and the exponent A depends directly on this relationship. This relationship is specifically included for the circular obstructions, and the slope (i.e., A) has a value of 1.619. Omitting the C_D and v_m relationship gives a slope or A value of 1.782 (see Equation 5-27). Neither of these values agree with the bridge pier values which are about 2.04 on average. Barring some systematic error in the current developments, it is assumed that the slope of 1.619 as in Figures 5-5 and 5-6 is correct. Omission of C_D gives the slope 1.782, and this value is also assumed to be correct. This latter value should match the bridge pier values; however, it does not. Drag coefficients and any variations in their values are implicitly included in the bridge pier formulas. Therefore, drag coefficients do not cause the slope difference, i.e., 1.782 vs 2.04. An alternative expression which includes the exponent A is given as the following Equation 5-42.

$$\frac{\Delta H}{m\delta} = K \left[\frac{v_m \delta}{v} \right]^A \quad 5-42$$

where K = some constant defined by δ , S_p/δ , and S_f/δ .

In this representation the A value in effect defines a rate of change of ΔH with v_m . Normal open channel flow would make A equal to 2.0 for flow in the rough range. This is close to the bridge pier values. Expressing the relationship in terms of the Reynolds number as is done throughout this development also specifies that A is dependent on δ . This is obvious from Equation 5-42. Therefore, the exponent A includes the effects of v_m , δ , and their variations in its value. As an example, the Wang data is considered twice in Table 5-26. Assumed A values of 1.619 and 2.0245 are used. The δ dependent B values are different for each of these slopes; however, the combined A and B values in either case serve reasonably well in defining the head loss. This implies that the exponent A considers more than just the rate of change of ΔH with v_m . Variations with δ are also included. A much better approach would isolate v_m and δ ; however, this has not been done here. It is assumed that the slope 1.619 is representative of the overall rate of change of ΔH with Reynolds number and that the appropriate δ dependent B value will be used with this slope.

With the exception of the Wang results, the values for the exponent C given in Table 5-26 match the current C value reasonably well. Nagler's and Yarnell's results for piers with a semicircular nose and tail are close to the 0.042 value. The twin cylinder pier values are also close. For both of these shapes, the Yarnell values are very close.

It was implied above that the coefficient B and the exponent A must be considered jointly. B partially accounts for the δ variation but primarily defines the magnitude of the losses. The values for B in Table 5-26 are derived for $\delta = 0.625$ ft. They are of course different for other δ values. In the last column of Table 5-26 are values of head loss per row calculated by using the values of A, B and C in the table. Velocity is selected at 1.0 fps, the drag coefficient for the first six rows is $C_D = 1.0$, the depth is 2.0 ft, S_1/δ is arbitrarily set at 3.0, and a square nose and tail are assumed for each of these first six rows. The last four rows use the shapes and drag coefficients given in the table. In all cases the head losses are greater for the square-end shapes than for the round-end

shapes. There is some variation in the data depending on the equation used, but the results are reasonably consistent. Note also that the head losses are roughly proportional to the drag coefficients as expected.

Yarnell considered three different δ values in his experiments. They were approximately 1.17 ft, 3.50 ft, and 5.00 ft. Assigning values to all of the variables but δ , the coefficient B in Equation 5-41 may be determined as a function of δ . As stated previously B varies with the natural logarithm of δ . Equations relating B and δ are simple when C_D is constant and are more complex when C_D is allowed to vary. The equation developed in this analysis relating magnitude of head loss and characteristic dimension δ is based on the experimental data and the basic logarithmic form. The numerical values in the equation fit the basic form to the experimental results.

All of the current experiments and the experimental results from other sources have considered only a limited range of δ values. Because of this it is necessary to extrapolate the current results for larger characteristic dimensions. This of course includes almost any natural setting for which structures are involved. Numerous means of expressing the current experimental data have been considered in an attempt to derive a more nondimensional form. Such a form was not found. It should be noted that the current data may be expressed in alternative forms which give the rates of change with spacing by a logarithmic expression rather than a power expression. Another alternative form factors out constants to express the head losses as a function of a linear term which contains the logarithm of the Reynolds number. The final form selected here is indicated by the equations presented earlier. They provide a more concise expression for head loss. Note again that the equations presented here have not specifically considered large δ values. Even though the equations provide reasonable values of head loss for large δ values, these head losses must be verified. This implies that the spacing function given here may require modification to specifically includes larger values.

Even though Equation 5-26 is expressed in a head loss per row form, this loss is dependent on the geometric properties of the entire obstruction system. These properties are specified by the spacing function. Note again that the head loss per row is directly proportional to the drag coefficient C_D . Thus any errors in the value of the drag coefficient are directly reflected in the calculated value of energy loss.

For natural systems which exhibit a high degree of uniformity in characteristic dimensions and spacing, the equations above are reasonably straight-forward and simple. When systems are not so uniform, it is necessary to use the equivalent system equations developed previously. In Chapter VI examples are given to illustrate the use of the equations.

CHAPTER VI EXAMPLES AND GUIDELINES

Bed Forms

The dimensionless equation derived for the bed forms is expressed in a slightly different form as Equation 6-1.

$$f'_b = \frac{2a}{L_b} \left[0.062 + 0.85 \left(\frac{2a}{d} \right)^{1.15} \right] \quad 6-1$$

where f'_b = friction factor for bed forms
a = one-half the bed form height
 L_b = length of one bed form
d = mean depth of flow over bed forms

Assume that dunes in a beach area are reasonably uniform and span a total length of 200 ft measured normal to the coast. Also assume that flood waters move directly inland at a depth of 3.0 ft relative to the average elevation of the dunes. For dunes which are 2.0 ft in height and 20.0 ft in length the friction factor would be $f'_b = 0.0595$. This friction factor is determined for a single bed form; however, it is applicable to the entire area covered by bed forms, i.e., 200 ft in this case. The length term in Equation 6-1 is the length of a single bed form and not the length of the area covered by bed forms. This latter length is considered in the Darcy-Weisbach equation.

All of the bed forms tested in the laboratory had crests which were normal to the direction of flow. Such conditions may not exist in the field. It is assumed but not proven that the length term in Equation 6-1 may be increased to reflect directions of flow other than normal. Note that this reduces the friction factor. It is also assumed that, within reason, the

average height and length of a group of bed forms may be used to calculate the friction factor. This may not be appropriate if the bed forms vary substantially in size. Individual consideration of the dunes may be required.

Note specifically that the friction factor varies with depth of flow. Since the tidal elevations vary with time, it is necessary to recalculate the friction factor as the conditions vary. This is not a serious problem since unsteady flow conditions are normally used in an evaluation.

Vertical Obstructions

The following Figure 6-1 represents a simplified residential area. Dwellings are about 1750 ft² in area and are centered on lots of about one-fourth of an acre. Right-of-ways A through E are 60 ft in width, and it is assumed that there are no fences or walls which act as barriers to flow. It is also assumed that the effects of small obstructions such as trees are negligible, and that the ground surface is horizontal. Skin friction losses will be neglected temporarily in this initial example. Depth of flow and velocity will be assumed to illustrate the computations for head loss. A north arrow is drawn in Figure 6-1 for reference.

Let flow around the obstructions in Figure 6-1 be from west to east at a mean depth of 3.0 ft and at a velocity of 0.5 fps. For this flow the characteristic dimension for all structures is 50.0 ft. Lots are set at 100 ft in width and 110 ft in length. Structures are 35 ft in length from front to back and are rectangular in shape as shown. The width of the flow area will be limited to the space between the center-lines of right-of-ways D and E. The flow length is bounded by right-of-ways A and C, center-line to center-line.

For the dimensions given above, S_f/δ may be computed and is equal to 2.15. This may be calculated as $b/n\delta$ where b is the total width of the flow region, n is the number of obstructions per row, and δ is the obstruction width, i.e., its characteristic dimension for the assumed direction of flow. S_f/δ may also be calculated as the average of seven flow areas with 100 ft of lateral obstruction spacing and one flow area with 160 ft of lateral obstruction spacing. This latter spacing is at the ends of the block where the two outer

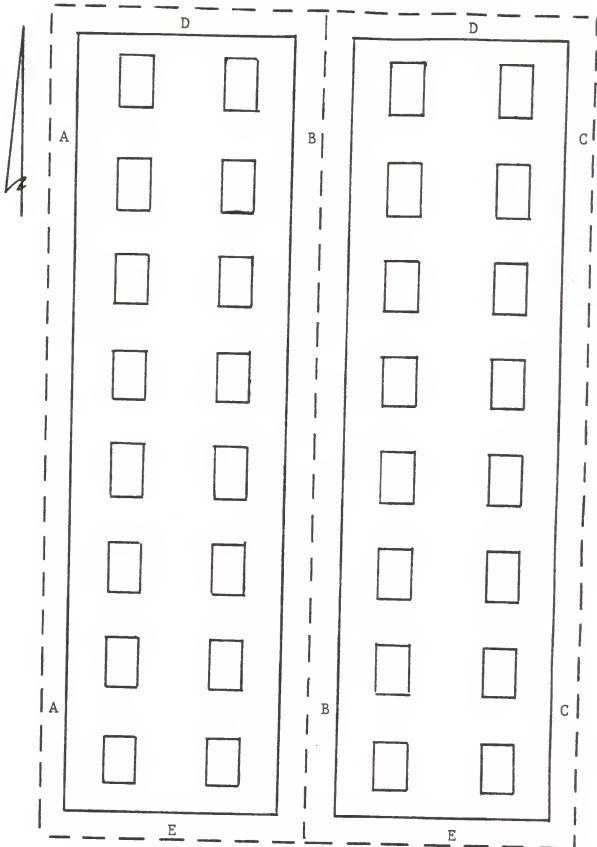


Figure 6-1: Simplified Residential Area
Scale: 1" = 100'

areas are combined into a single flow area as mentioned previously. The dimensionless longitudinal spacing S_l/δ may be calculated as $L/m\delta$ where L is the defined length of the flow region, m is the number of rows, and δ is again the characteristic dimension. Note that the length of the obstructions in the direction of flow is not used as the characteristic dimension. Here S_l/δ is equal to 2.80. This may also be calculated as the average of two spaces with 110 ft of longitudinal spacing and two spaces with 170 ft of longitudinal spacing. Note specifically that the S_l/δ value of 2.80 includes the assumption that upstream of the first row of obstructions are similar obstructions which affect the first row in the same manner that rows two and beyond are effected. This will be considered in more detail later.

Using δ equal to 50 ft and the dimensionless spacings above gives a value of the spacing function $H = 29.9425$. Using $g = 32.174 \text{ ft/sec}^2$, $\nu = 1.4 \cdot 10^{-5} \text{ ft}^2/\text{sec}$., and the values given previously, the coefficient ratio $\eta = 36.2924$. Note that this value is larger than might be expected. Letting $C_D = 2.0$ gives a friction factor $f'_D = 0.7234$. Using the Darcy-Weisbach equation directly gives an obstruction head loss $\Delta H_v = 0.5247 \text{ ft}$. The head loss per row is $\Delta H_v/m = 0.132 \text{ ft}$ or $\Delta H_v = 0.5247 \text{ ft}$ since the number of rows $m = 4$. Thus the energy loss per unit weight of fluid is 0.5247 ft in a flow length of 560 ft. The energy grade line would have an average slope of $0.37 \cdot 10^{-4}$. All of the values here assume a mean velocity of 0.5 fps and a mean depth of 3.0 ft. The depths at the upstream and downstream flow boundaries, respectively, would be approximately equal to the mean depth plus and minus one-half the head loss. Exact depths would need to consider the velocity heads.

Only the obstruction losses were considered above, and skin friction losses were neglected. Using an approximation for the roughness and the assumed mean velocity of 0.5 fps, a loss due to skin friction will be calculated for comparison. If the flow region has equivalent sand roughness $k = 0.5 \text{ ft}$, then flow is in the Manning range. The friction factor in this case would be $f'_s = 0.01617$ where the mean depth of 3.0 ft is used as hydraulic

radius. The Darcy-Weisbach equation gives a skin friction loss of 0.0117 ft. The actual skin friction loss would be slightly larger than this if the alternative velocity mentioned earlier was used. Using either velocity, however, the skin friction loss is small in comparison to the obstruction loss. The value of equivalent sand roughness used here may or may not be a typical value. Different roughness values and obstruction densities could completely change the relative magnitudes of the two losses. These values are simply for comparison.

The example above is rather simple since mean depth and velocity were both specified. A more practical example would specify piezometric heads at the flow boundaries and require determination of the discharge. This may be done by solving for mean velocity v_m in the Reynolds number. The amount of iteration in this procedure depends on the assumptions made concerning the averaging of the velocities. To illustrate this let the upstream and downstream piezometric heads in the previous example be 3.0 ft and 2.0 ft, respectively. Recall that the ground surface was considered to be horizontal so differences in piezometric head are equivalent to differences in depth. The equation expressing conservation of energy is given as Equation 6-2. Subscripts one and two imply upstream and downstream rectangular sections, respectively.

$$\left[d_1 + \frac{\alpha_1 Q^2}{2gb^2 d_1^2} \right] - \left[d_2 + \frac{\alpha_2 Q^2}{2gb^2 d_2^2} \right] = \frac{m\delta C_D \left[\frac{v_m \delta}{v} \right]^{1.619}}{(6^{22.46} d^{1.87})^\omega} \quad 6-2$$

- where
- d = depth
 - α = energy coefficient (assumed 1.06)
 - Q = discharge
 - g = acceleration due to gravity (assumed 32.174 ft/sec²)
 - b = obstruction-free width of flow area (860 ft here)
 - m = number of rows in flow area (4 here)

δ = characteristic dimension (50 ft here)

C_D = drag coefficient (assumed 2.0)

v_m = mean velocity

ν = kinematic viscosity (assumed $1.4 \cdot 10^{-5}$ ft²/sec)

e = 2.71828 . . . = base for natural logarithm

$$\omega = \frac{\left(\frac{S_t}{\delta} - 1.0\right)^{0.042}}{\left(\frac{S_t}{\delta}\right)^{0.0049/(\delta^{0.743})}}$$

S_t = transverse spacing

S_l = longitudinal spacing

Mean velocity v_m in the above equation is not only a sectional mean but a longitudinal mean as well. This mean may be defined by a simple or a more complex procedure. Recall that integration of the water surface profile was used in the development of equations to define a mean depth d_m , and v_m was defined using d_m calculated by this more complex approach. In as much as velocity heads are generally small, a simple procedure should be sufficient here. Let d_m equal the average of depths d_1 and d_2 , and let v_m be equal to the discharge Q divided by bd_m . Here b is the width of the unobstructed flow area, i.e., 860 ft. Equation 6-2 would then have a single unknown, namely Q . In this example $Q = 1593$ cfs. Neglecting the velocity heads completely would give $Q = 1601$ cfs. Mean velocity here would be 0.74 fps at a mean depth of 2.5 ft.

The examples above represent only two of many possible examples. When compared to normal open-channel flow analysis, the first example is similar to a standard-step-method, i.e., head losses are computed for a fixed length of flow area. The second example is similar to a direct-step-method which considers a specific energy loss. It differs from a direct-step-method, however, since velocity or discharge is unknown rather than length of the flow area. One primary difference in the computations concerning obstruction

losses and normal open-channel losses is the friction factor. Friction factors in normal open-channel flow are independent of velocity; however, they are highly dependent on velocity for vertical obstructions.

Highly uniform systems with convenient directions of flow like the preceding examples imply that the determination of appropriate values for the apparent obstruction density or the dimensionless spacing is not too complicated. Typical real situations may or may not be as simple or have such high uniformity. The theoretical and experimental equations indicate that the friction factor f'_D is directly proportional to the equivalent obstruction density P_D . P_D is easy to compute except in those cases where the direction of flow makes determination of the characteristic dimension questionable. An example would be a flow directed at the corner of a rectangular building. In this case the width of the projection of the building in the direction of flow provides the most appropriate characteristic dimension. The drag coefficient used should be consistent with this dimension and direction of flow. Non-convenient directions of flow may also make determination of appropriate dimensionless spacing values more complicated. The exponent ω in the experimental equation depends directly on the dimensionless spacing values. The graphs and equations given previously which were used to develop the ω expression should indicate the importance of appropriate dimensionless spacing values, especially S_1/δ . Because of this it is important to note certain observations concerning P_D , S_1/δ and S_2/δ .

Each of the laboratory test groups considered a single pipe diameter, and it should be expected that as longitudinal spacing is increased a spacing would be reached where an upstream obstruction had little or no effect on the obstruction downstream. The maximum dimensionless longitudinal spacing considered was approximately 50, and the spacing function appeared to approach a constant value around 50 or slightly beyond. The rate of change for the spacing function, however, is expressed in terms of a power formula for which there is no upper limit to the effects of spacing on the equation. Physically of course there is an upper limit. It will be assumed, based on the plotted spacing function, that the

effective maximum dimensionless longitudinal spacing is 100. Beyond this point upstream obstructions have no effect on those downstream. Theoretically the minimum dimensionless longitudinal spacing is 1.0, although smaller values are possible for noncircular shapes. Recall that the dimensionless longitudinal spacing was considered to include the combined effects of sheltering of the obstructions and the decay of turbulence with increased longitudinal spacing. Using the assumed maximum value of 100, it is reasonable to assume that two obstructions of 50 ft in diameter and about a mile apart have no effect of each other.

Values of S_l/δ therefore range from about 1.0 to 100, and every obstruction has a representative value of S_l/δ . Although S_l/δ plays only a minor role when the δ values are large, it is necessary to provide a consistent method which may be used to assign to each obstruction a representative value of S_l/δ . The following method is suggested. Assign to every obstruction i of characteristic dimension δ_i an isosceles triangle i of base length δ_i and altitude $100 \delta_i$. The tip of triangle i points in the direction of flow. The base of triangle i is positioned at the center of obstruction i . Any downstream obstruction j enclosed or intersected by triangle i is influenced by obstruction i . Let the center-to-center longitudinal distance between obstruction i and j be L_{ij} and compute L_{ij}/δ_i . This value applies to obstruction j . Since any obstruction j may be influenced by more than one obstruction i , select the minimum value of L_{ij}/δ_i for that obstruction j . The S_l/δ value representative of the entire system is the weighted mean of each obstruction's minimum L_{ij}/δ_i value. The weighing factor should be each obstruction's δ value. An obstruction with no obstruction upstream would have a value of 100 for its S_l/δ value.

The dimensionless transverse spacing S_t/δ provides an inverse measure of the energy dissipation required when water is forced to flow between obstructions, i.e., to flow into a contracted section. Contraction and expansion losses in the main body of the flow cannot be separated from the eddy losses produced by the obstructions which form the contraction. Since both effects are combined, it cannot be assumed that there is an

upper limit to the transverse spacing portion of the spacing function. It should also be noted that the S_T/δ portion of the spacing function provides an averaging effect on the head losses per row. Consider a single, small obstruction in a channel of 100 ft in width. A certain head loss is produced. Now consider the same obstruction in a channel of 200 ft in width with the same conditions of flow. The difference in the widths of the channels should cause an almost insignificant difference in the actual energy lost in the flow around the obstruction. The S_T/δ values, however, differ by a factor of 2. This gives an energy loss different from that of the initial channel. This difference represents an averaging of the actual loss over two different widths. This implies that all flow areas must have boundaries to the left and right of the flow. These boundaries may be real as in the laboratory flume, or they may be imaginary as in the previous examples. Imaginary boundaries should be consistent with the system considered and the desired meaning of head loss. Is the head loss for a specific area required or is some average over a larger area required?

S_T/δ may also be considered as the ratio of two system lengths. For a given row of obstructions it is the total width of the flow area (obstructions and flow) divided by the total width of the obstructions in that row. Total width of obstructions here implies the sum of the row's characteristic dimensions. Here again the left and right boundaries of the flow region must be defined. In the previous paragraphs a representative dimensionless longitudinal spacing was considered for each obstruction. A representative dimensionless transverse spacing may also be defined to compare one obstruction relative to another. For any given transverse row of obstructions the weighted mean of the representative S_T/δ values is equal to the ratio of lengths mentioned above. Characteristic dimension δ is the weighting factor. Since a given row may contain one or many obstructions, three different definitions are required.

For a single obstruction between the left and right boundaries of the flow, the representative S_T/δ value would be the defined width divided by the obstructions

characteristic dimension. For an interior obstruction not adjacent to either boundary, the following Equation 6-3 may be used.

$$\frac{S_t}{\delta} = \frac{2(\delta + S_{tL} + S_{tR}) - (\delta_L + \delta_R)}{4\delta} \quad 6-3$$

- where S_t/δ = representative dimensionless transverse spacing
- δ = obstruction characteristic dimension
- S_{tL} = center-to-center spacing between the obstruction considered and the obstruction to the left
- S_{tR} = center-to-center spacing between the obstruction considered and the obstruction to the right
- δ_L = characteristic dimension for obstruction to the left
- δ_R = characteristic dimension for obstruction to the right

A particular transverse boundary may be called left or right as desired, but the use should be consistent. For an obstruction adjacent to either boundary but not to both, the following Equation 6-4 may be used.

$$\frac{S_t}{\delta} = \frac{\delta + 4S_b + 2S_{tRL} - \delta_{LR}}{4\delta} \quad 6-4$$

- where S_t/δ = representative dimensionless transverse spacing
- δ = obstruction characteristic dimension
- S_b = transverse distance from the center of the obstruction considered to the adjacent boundary
- S_{tLR} = center-to-center spacing between the obstruction considered and the more interior obstruction (left or right)
- δ_{LR} = characteristic dimension of more interior obstruction (left or right)

For any transverse row of obstructions a weighted mean of the representative S_t/δ values may be used to consider all or any non-segmented portion of that row. When more than one row or the entire system is considered, it may be necessary to consider the losses on a row by row basis. The exponential and power formula relationship between

energy losses and S_t/δ values implies that an arithmetic mean of S_t/δ values for two or more rows may not be accurate. This especially applies when one or several of the rows have very different values of S_t/δ . Note that all of the laboratory tests considered uniform systems for which the equations apply to a single row or to many rows. Whether an entire system is considered as a whole or by sections must be decided based on the system.

Consider the initial example representing a simplified residential area bounded by right-of-ways A,C,D, and E. Remove all dwelling except a single row in either of the two blocks, and neglect skin friction losses. The width of the flow area is defined to be 860 ft and the length is 560 ft. S_t/δ may be calculated as before and still has a value of 2.15. S_r/δ now has a value of 11.2. This value and the new value of P_D may be used with the same equations to give the new energy loss or other variables. These values would be correct if at a distance of 11.2δ upstream of the row within the system there is another similar row which is not within the system. The laboratory equations are developed in such a manner that S_r/δ must represent the actual physical spacing of the rows. An exception is the assumed maximum value of 100. Computing ΔH for the single row here gives $\Delta H_v = 0.1327$ ft when the same values of velocity and the other unchanged variables are used. Note that this value is very close to the head loss per row calculated previously and is slightly larger as expected. For the assumed maximum S_r/δ of 100, the head loss is 0.1350 ft.

At the other extreme assume that the number of rows of dwellings is increased to the point that the dwellings actually make contact and isolated channels are formed. This condition is not considered by the current equations, and any calculations of this nature would obviously be in error. It is estimated that the minimum possible value of S_r/δ for which the current equations may be used is approximately 2.0. The smallest value considered by any of the laboratory tests was about 4.0. Small values of dimensionless

longitudinal spacing suppress the contractions and expansions in the flow between adjacent rows.

The experimental equations contain the equivalent obstruction density P_D . An alternative definition of P_D was used during the development of the equations; however, this definition should not be used with the experimental equation. In general, P_D will not equal the reciprocal of the product of S_V/δ and S_T/δ . Only when the obstructions are arranged in symmetric transverse and longitudinal rows is this alternative definition valid.

As a final example the use of the equivalent system of circular obstructions will be considered. The theoretical equations given previously may be used to determine the number and size of these equivalent obstructions. Recall that those equations did not consider the dimensionless spacings S_T/δ and S_V/δ . Because of the importance of these two variables, especially S_T/δ , careful consideration should be given to those values and just how the field system is analyzed. Some of the options for S_T/δ and S_V/δ will be considered for the following assumed field conditions.

Figure 6-2 is a scale drawing of what will be considered a typical ocean front arrangement of structures. Both large and small structures are included, and the boundaries of the system are arbitrarily defined. To illustrate the use of the theoretical equations, this system of structures will be analyzed strictly according to the equations and the assumptions made during their development. Note now that this is not the best approach. This system should be divided into two subsystems, and each of these should be analyzed in a manner similar to the procedures given below for the entire system. A north arrow is drawn in the figure for reference. Flow is from east to west at an assumed depth of 3.0 ft and at a velocity of 1.0 fps. Minor obstructions are omitted in this analysis, and the ground surface is assumed to be horizontal. Note that a designation of surface roughness is not required by the theoretical equation. This is because those equations do not include skin friction losses in the head losses calculated by the determined values of equivalent diameter and equivalent number of obstructions. Skin

friction losses must be considered separately and added to the obstruction losses. Table 6-1 gives data pertaining to the ten obstructions indicated in the figure and denoted A through J.

For the natural system $C_{Dmn} = 1.93$, $\delta_{mn} = 95.9$ ft, and the number of obstructions $N = 10$. The distribution factor $\lambda = 0.9885$. Recall that the shape factor C_n is equal to the obstruction's actual plan area divided by the square of its characteristic dimension. These values are calculated and are given in the table. Shape factors are used to obtain the modified characteristic dimensions δ'_n also given in the table. Assuming that the equivalent characteristic dimension is the same order of magnitude as δ_{mn} implies a very large Reynolds number. From Figure 2-2 it may be seen that the drag coefficient is small, and a value of 0.37 will be assumed here. The shape factor for the equivalent circular obstruction is $\pi/4$. The mean and standard deviation for the distribution of δ'_n values may be calculated and are 76.7535 ft and 55.8361 ft, respectively. Note that this relatively large standard deviation occurs because of the large spread in the values of characteristic dimension and area. Calculations using the values above give the equivalent characteristic dimension $\delta_e = 23.198$ ft. This is smaller than expected due to the small drag coefficient. The number of equivalent obstructions is $N_e = 213.147$. This is much larger than expected and is again due to the small drag coefficient. Note that the drag coefficient for characteristic dimensions of 95.9 ft and 23.2 ft are about the same. If this were not the case the number of obstructions and their size would need to be determined by iteration based on the value of the drag coefficient.

It was stated previously that the total area of obstructions in the natural and equivalent systems were equal. Calculating these values indicates an error of a few percent. This error occurs because of the small number of obstructions and the probability relation used in the development. The total area A_L of both systems is the same and is 425,000 ft². This implies an area of 1993.93 ft² for each of the 213.147 equivalent obstructions. Note that it is not necessary to round this off to a whole number. Each

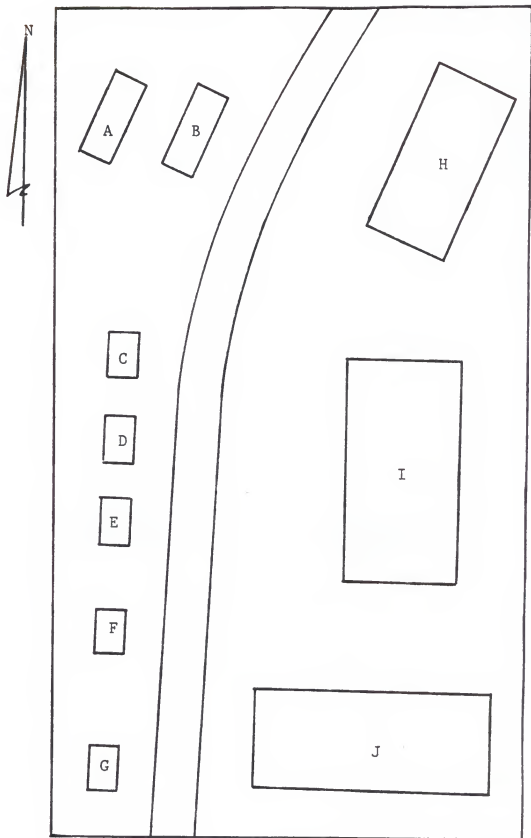


Figure 6-2: Simplified Ocean Front Area
Scale: 1" = 100'

Table 6-1. Structure Data

Structure	Area (ft ²)	δ_n (ft)	C_{Dn}	C_n	δ'_n
A	3,150	95	1.8	.349	56.122
B	3,150	95	1.8	.349	56.122
C	1,450	50	2.0	.580	38.079
D	1,450	50	2.0	.580	38.079
E	1,450	50	2.0	.580	38.079
F	1,450	42	2.0	.822	38.079
G	1,450	42	2.0	.822	38.079
H	16,200	200	1.8	.405	127.279
I	32,200	230	2.0	.609	179.488
J	25,000	105	1.9	2.268	158.129

obstruction has a diameter of 23.198 ft. Under the assumptions initially made when the equations were developed, equivalent obstruction density P_D was the key factor and friction factors and head losses were proportional to P_D . This is still correct but the dimensionless spacings modify this proportion. Based strictly on P_D , a uniform distribution of obstructions is sufficient. This would place each obstruction in the center of a square area which was 44.653 ft on each side. Such an area would give $S_1/\delta = S_l/\delta = 1.925$. Using these values to compute an energy loss would give a loss per row of 0.0548 ft. There are however 14.6 rows for which the total head loss would be 0.801 ft. This loss would be approximately correct if the natural system had obstructions which were reasonably uniform in their distribution. They do not need to be uniform in size. A more correct head loss for this system is roughly twice the head loss calculated above. This may be determined by direct application of the experimental equation to the actual field area. The area should be divided into two subareas - one with obstructions H, I, and J and the other with the remaining obstructions. This analysis is reasonably simple and will not be given here. The procedure was considered in the first two examples.

The value of S_1/δ used with the equivalent system should match the actual field value of S_1/δ as much as possible. A great deal of flexibility is included in the S_l/δ value because of its minor effect on the results. The field area and the equivalent area do not need to have the same number of rows. Because of this the S_1/δ value of the two areas may be set exactly equal, and the number of rows adjusted according to the density P_D .

As a final comment it should be noted that conditions of supercritical flow between obstructions have not been considered during this development. Therefore high velocities and/or severe contractions which significantly choke the flow should be approached with caution. Small values of S_1/δ have been considered here, but not simultaneously with high velocities. The previous example and the assumption of uniform conditions stretches the limits for which the equations developed here are intended. The conditions considered

during this development should serve as a guide to application of the equations, and steady or quasi-steady flow conditions must be assumed.

CHAPTER VII

CONCLUSION

Dunes in a beach area are basically formed by the wind and not by flowing water. When compared to bed forms in an inland waterway, dunes differ in shape and in most cases size. In spite of these differences it is assumed that dunes and bed forms are similar in their reaction with flowing water, and that methods developed for bed form friction factors may be applied to dunes. In particular it is assumed that Equation 5-4 may be used to determine a friction factor for the flow of water over dunes.

The shape for the bed forms used in the current experiments represents an average of natural bed form shapes. This shape was determined by other research as noted in the literature review. The use of an expansion loss equation such as the Carnot equation has also been suggested by others; however, the form of the current expression is different. Energy coefficients and a dimensionless, numerical coefficient ϕ have been included in this analysis.

Using the experimental values for energy coefficients, a ϕ value could be determined which would make the theoretical equation match the experimental results. This ϕ value would be correct for the current tests; however, it is doubtful that this value could be used in general. It is therefore not specified. An extended series of tests with different bed form shapes may produce ϕ values as a function of shape; however, this seems pointless in view of the variety of natural systems and how well these systems may be defined in space and time. A reasonably simple expression which requires a small number of parameters seems to be the best approach. This is provided by Equation 5-4. Energy

coefficients and ϕ values are implicitly included in the equation, and only length and height of the bed forms are required. Flow depth is a variable.

The results of the vertical obstruction tests may be expressed in different forms. Equation 5-25 is an expression for the friction factor, and Equation 5-26 is an expression for head loss per row. Both of these expressions are based on the limited test data and outside information available. Large characteristic dimensions were not directly considered. Equations 5-25 and 5-26 may therefore be improved by direct consideration of larger dimensions.

Perhaps the most significant relationship is expressed by Equation 5-18. This relationship may be converted into a head loss per row form which gives the losses as a product of characteristic dimension, drag coefficient, Reynolds number to the power 1.619, and a modified spacing function. This spacing function acts as a proportionality constant to define the magnitude of the losses. The spacing function developed in this analysis also defines the magnitude of the energy losses; however, it is extrapolated to consider dimensions not proven by experiment. Additional testing could improve the current spacing function, especially the lateral variation. Equation 5-18 does however show that head losses are directly proportional to drag coefficients, or in effect, to the drag forces. Energy losses are also proportional to velocity and characteristic dimension, but these terms are given to a power other than unity in the proportionality.

The equivalent system of obstructions has been developed to consider irregular systems which are not subject to direct application of the equations. It is assumed that average values of the parameters are suitable for evaluation of the energy losses. This assumption is questionable for small systems such as the example considered in Chapter VI; however, the average values should provide very good results for large systems.

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John Dorman was born on January 16, 1947, in Jacksonville, Florida. He graduated from high school on June 4, 1965, in Moore Haven, Florida. After three years in the United States Army, he entered college at the University of Florida in the fall of 1968.

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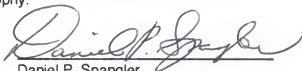
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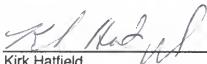
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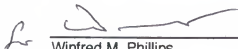
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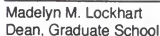


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